Optimal portfolio choice with stock market entry costs and human capital investments : a developing country model

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Abstract

This study estimates the magnitude of the implicit stock market entry cost, which may explain the observed level of non-participation in developing countries. Employing a two-generation, multi-period model with income risk, borrowing constraints and flexible human capital investment options in the offspring, the study numerically computes the possible improvements in consumption and human capital if households participate in the financial markets. The study extends the portfolio choice literature by endogenizing human capital investments and compares the optimal portfolio choice graph of a parent with that of a single worker. Results show that a parent's portfolio should contain more bonds when educational investments are inflexible and more stocks when they are flexible. Educational investments can act as an additional handle of adjustment against financial risk. Lower asset levels of a parent also tend to increase stock holding percentage. The study contributes to the portfolio choice literature as well as the financial inclusion literature on developing countries.

Keywords: `Portfolio choice, Stock market, Human capital, Finance, Dynamic programming`
JEL classification: G11; N35

1. Introduction

Several studies have established a correlation between banking & stock market development and the growth performance of countries[1] (Beck & Levine (2004); Naceur & Ghazouani (2007); Fink et al. (2009); Ruiz (2018)). One of the arguments is that efficient delivery of financial services through well-integrated banks and stock markets enables an economy to grow as well as help its poor integrate with the rest of the economy (King & Levine (1993); Bencivenga et al. (1995); Demirguc-Kunt & Levine (2009); Esso (2010), Van Rooij et al. (2012)). However, studies like Haliassos & Bertaut (1995); Campbell (2006) point out that a large number of households do not participate in the stock markets. In countries like India, only 6.5% of rural households and 7.3% of urban households invest in the stock markets while most of the other investments are held in consumer goods, jewellery and other instruments (Shukla (2010)). A SEBI (2015) survey report states that only 25% of even college educated households participate in the stock

[1] Ductor & Grechyna (2015) and Ergungor (2008) highlight non-linear relationship between financial development and growth, and the importance of financial structure

markets. Some of the reasons for non-participation are attributed to costs that deter entry into the stock market. Vissing-Jorgensen (2002) categorizes the costs into three categories: fixed entry costs, transaction costs, and per period trading costs. Given the observed complete non-participation of a large section of households in developing countries (as opposed to intermittent participation), a strand of literature identifies low levels of financial literacy and lack of information about the stock markets as one of the key entry barriers to stock market participation (Lusardi & Tufano (2009); Lusardi et al. (2010); Rooij et al. (2011)).

While the positive correlation between stock markets and economic growth is well documented, the specific benefit of stock market access on households' human capital outcomes and consumption in the developing world is not well researched. A probable reason could be the lack of microeconomic data on stock market participation in developing countries. However, consumption and human capital decisions are typically made at the household level under the influence of various forces and constraints, and therefore it is important to assess the impact of stock market participation at the household level.

In the general absence of data on stock market participation in developing countries, this study's contribution is the introduction of a multi-period, two-generation model with endogenous human capital acquisition and income risk, to assess the likely effect of financial market participation on households' educational investments and consumption. In doing so, the study examines three representative parent-child households with different income levels of the household heads and numerically solves the optimal allocation of stock in the savings portfolios of the household heads over their lifetimes. The simulations of the study also attempt to estimate the magnitude of the fixed entry cost that would explain the degree of non-participation captured in the surveys. The study contributes to the portfolio choice literature by introducing a unique, developing country model where the offspring's education can be flexibly adjusted by the parent to augment the household income in times of low parental income.

The presence of an offspring plays a vital role in the determination of household savings because of the child's higher consumption and educational needs. A child's presence adds an additional event in the parent's life-cycle, dividing it into two broad phases: when the parent and the child are together and when the adult offspring lives independently. The risk appetite of the parent is likely to be different in the different phases, and the study aims to understand the adjustments in a parent's asset portfolio in the various stages of his/her life as a response to risk.

The primary sources of risk in an economy are the labour market shocks and the stock market shocks. The presence of aggregate shocks to the economy makes the labour income process correlated with the dividend process. Baxter & Jermann (1995), Campbell (1993) and several other studies document the presence of a strong correlation between labour market returns and stock market returns. Hence, in the event of high risk (strongly correlated labour and stock markets) a parent's portfolio allocation with a dependent child is likely to adjust differently from that of a single individual.

The literature on portfolio choice started with the seminal work of Samuelson (1969) and Merton (1969), where they posited that individuals should hold a constant fraction of their assets in risky stocks at all points in their lifetimes. It was later argued by Cocco et al. (2005), that the presence of non-tradable labour income (which behaves like a riskless asset) under borrowing constraints creates a strong incentive for individuals to rebalance their portfolio towards stock in the earlier periods of their life when future labour income is abundant, and assets are low. The share of stock in the savings portfolio should gradually fall as individuals approach retirement as then their endowment of future labour incomes are lower, and wealth levels are higher.

Recent studies by Love (2010) and Hubener et al. (2016) have included children in the parents' optimization problem to model the impact of demographic shocks like marital transition and children on portfolio choice. They find that children play an important role in portfolio choice because of higher consumption needs and college costs.

While Love (2010) and Hubener et al. (2016) model education costs as a lump sum payment of permanent income, this study explicitly models a parent's investments into his/her offspring's human capital by considering downward altruism as well as parental valuing of offspring's education as the two motivators of parental investments.

In doing so, this study considers a two-generation, finite time-period life-cycle portfolio choice problem of an altruistic parent facing the challenges of consumption, educational needs, and retirement savings in the presence of borrowing constraints. The specific focus of the study is to understand the mutual interaction between human-capital investments and stock market investments as an adjustment mechanism to risk.

The results of the study make some valuable contributions to the development literature on financial inclusion as well as the life cycle portfolio choice literature. While financial market participation (stock & bond) is seen to improve the human capital outcome of the middle-income household by 20%, the lowest income household's investments in its offspring's human capital remain completely inelastic to the level of financial access. This category is not able to take advantage of the higher return on savings made possible through financial market access (bond & stock) mainly due to the low asset levels in the earlier periods when borrowing constraints bind. A combination of income support and bond market access is an efficient way of pulling this category out of the intergenerational poverty trap.

The study estimates that an implicit entry cost of about 155% of the minimum wage (1100 dollars) explains the level of non-participation witnessed in the Indian stock markets as captured by surveys. Simulations also reveal that when faced with fixed entry costs, the households that participate, prefer to enter the stock market either before the education of the offspring starts or after the offspring's education is over. It is not optimal to enter while the human capital investments are in progress.

The study also reveals some interesting deviations in a parent's portfolio choice graph as compared to a single worker. In high-risk situations (risky stocks or highly risky labour income or high correlation between income and stock market shocks), a parent with inelastic human capital investment options, shifts his portfolio more towards bonds as compared to a single worker as a risk mitigating strategy. The study argues that a dependent offspring reduces a parent's risk appetite by altering the curvature of the value function. However, when human capital investments are elastic, the study also finds an interesting counter-force, that the ability to adjust human capital investments as a response to financial risk, enables a parent to handle more financial risk. Human capital investments add another margin of adjustment for the parent, which increases his/her risk appetite, thus increasing the fraction of stock in the household portfolio. Also due to higher expenditures on a dependent offspring's consumption and education, the asset levels of a household are lower as compared to a single worker, which tends to tilt the portfolio towards the risky stock for more extended periods of time. Therefore, the presence of a child adds two additional influences on a household as compared to a single worker: lower asset levels due to higher expenditures on the child leading to a portfolio tilt towards the more risky stock; flexibility in human capital investments can add an additional margin of adjustment against financial risk, thus leading to more stock holding. The optimal portfolio choice is the trade-off of these competing forces. The portfolio choice graph remains invariant with stock market entry costs. Higher entry costs, however, delay the point of entry into the stock market.

This study integrates into the existing and growing literature on various aspects of life cycle optimization and portfolio choice like precautionary motives (Haliassos & Michaelides (2003)), effect of taxes (Dammon et al. (2001)), the role of life annuities (Horneff et al. (2008), Horneff et al. (2010), Milevsky & Young (2007)), asset pricing (Constantinides et al. (2002), Storesletten et al. (2007), Curatola (2017), Jacoby et al. (2019)), housing as an asset (Cocco (2005), Hu (2005), Yao & Zhang (2005), He et al. (2019)) and the effect of momentum on portfolio selection (Wu et al. (2017), Moskowitz et al. (2012)).

The rest of the study is organized as follows: section 2 describes the environment in detail while sections 3 & 4 discuss the parameter values and simulation outcomes for the three representative households. Sections 5 & 6 present the optimal portfolio choice graphs of the heads of households as well as sensitivity analysis for different parameter values.

2. The Environment

This study aims to solve the life cycle portfolio choice problem of a parent in the presence of entry costs for stock market participation and endogenous investment options in the human capital of the offspring. In doing so, the study also computes the improvements in consumption levels and human capital investments due to financial market participation. For that purpose, the study formulates a two-generation, seventy time-periods' model with per-period constraints on human capital & financial investments. The study assumes two drivers for parental investment in the offspring: downward altruism & parental valuing of the

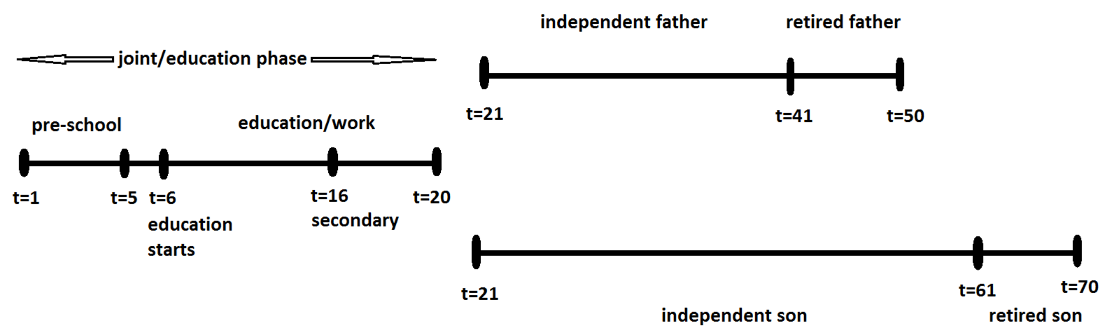


Figure 1: Time-line : Two-generation, seventy time-periods' life-cycle

offspring's education. In the model, the production function for human capital is assumed to have two inputs in the form of education time and books, and wages and asset returns are considered to be stochastic.

This study works with a single offspring (son) single parent (father) set-up. The model is gender-neutral, and the focus is primarily on the transfer of resources from the older generation to the newer. Time, which is discrete, starts from $t = 1$ and continues up to seventy. The father and the son have a twenty year age gap between them, and at $t = 1$ the offspring is assumed to be an infant aged one. In the first twenty periods of the model, the father and the son live together as a single household where the father is responsible for the consumption and educational needs of the son. After that, the grown-up son separates and lives independently.

In the pre-school stage (first five years) the parent is assumed only to bear the consumption expenses of the child. Even though the actual consumption of the child would be much lower than the parent's consumption, the study assumes equal consumption expenditures for the parent & the offspring to account for early child care and health-related expenses of the child. Since one of the aims of this study is to understand the impact of financial market participation on the educational attainment of the offspring through formal schooling (by giving up the farm, household or market work), the effect of parental investments in the pre-school stage on the human capital outcome of the child is ignored.

From $6 \leq t \leq 20$, the parent is responsible for the human capital acquisition of the child. Under extreme cases, the parent may also alternatively make the child work to augment the family income. The human capital acquired by the son in these periods determines his income level as an independent adult. The son starts his independent life at $t = 21$, and the independent parent and the adult offspring are assumed to be economically independent. Age 60 ($t = 40$) is the last period of the father's working life, and the father dies at age 70 ($t = 50$). The offspring (son), who begins his independent life without any assets, retires at the age of 61 ($t = 61$) & dies at the age of 70 ($t = 70$). The study doesn't include further generations.

Offspring 's human-capital

From time, $6 \leq t \leq 20$, the parent takes the human capital decisions for the offspring. The son has a unit endowment of time every period. The parent may decide to make the child work to increase the family income or send the child full time to school or choose an intermediate combination. The fraction of the unit amount of time the child is engaged in schooling at time t is denoted as λ_t while the amount of non-time educational inputs like books invested in the child by the parent is referred to as b_t. With the human capital acquired in the $6 \leq t \leq 20$ time periods, the son starts his independent adult life from $t = 21$. His adult periods' income is dependent on the amount of human capital acquired when the father-son duo live together as a joint household.

The parent is assumed to be borrowing constrained in each period as he is not allowed to borrow from his future labour income as there is income risk and also cannot borrow against the offspring's future income because of moral hazard issues.

The father's level of human capital is assumed to stay constant over time at $H_{f,t} = H_{dad}$. The son's human capital production follows a standard two input Ben-Porath (1967) function. The human capital produced in any period t is a function of ability θ, time and books invested in education (λ_t and b_t), and the current level of human capital $H_{s,t}$ of the son. The rate of depreciation is denoted by δ.

$$
\begin{aligned}
H_{f,t+1} = H_{f,t} &= H_{dad} \\
H_{s,t+1} = f(\theta, \lambda_t, H_{s,t}, b_t) = \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} &+ H_{s,t}(1-\delta) \\
min(H_{s,t+1}) = \bar{H} &= 1.0
\end{aligned}
\tag{1}
$$

The human capital level is assumed to have a lower bound of one ($\bar{H} = 1.0$). As the study doesn't model the effect of nutrition on human capital, it is assumed that the human capital of the offspring stays constant at the base level $\bar{H}$ till the beginning of age 6 (pre-school is supposed to have no formal education) from when human capital acquisition starts. Since the parent solves the optimization problem at $t = 1$, and the parent assumes that the offspring will separate at $t = 21$, the parent only takes formal education up to college[2] into consideration. The study stays away from modelling On-the-Job Training (OJT).

The maximum possible time input into education at any time period t is referred to as $\lambda_t = \lambda_{max}$ where λ_{max} is zero in the pre-school years and is equal to one in the other time periods. There is also an upper cap on the amount of non-time inputs (books) that can be invested in schooling at any time period t, which is referred to as $b_t = b_{max}$. The presence of b_{max} makes the foregone human capital acquisition in the earlier periods partly irreversible by precluding infinite investments in books in any chosen later time period. b_{max} is assumed to be a function of the income bracket the household falls under as good school resources are generally inaccessible to poor families due to distance and discrimination reasons.

[2] refers to three years in college & twelve years in school

The maximum possible human capital the offspring can possess at the beginning of any time period $t+1$, corresponds to the maximum educational inputs (λ_{max}, b_{max}) being used for the son's human capital acquisition in the previous time period t.

$$H_{s,t+1}(\lambda_{max}, b_{max}) = \theta(\lambda_{max})^{aa}(H_{s,t})^{bb}(b_{max})^{cc} + H_{s,t}(1-\delta) \tag{2}$$

Like Jafarey & Lahiri (2002) the father is assumed to gain explicit period-by-period utility from human capital inputs (edu-time λ_t & books b_t) he can afford for his son's human capital production. The form of the per-period utility the father gains is of the following functional form:

$$u(\lambda_t, b_t) = pen.\Big(H_{s,t+1}(\lambda_t, b_t) - H_{s,t}\Big) \tag{3}$$

where $pen = 1.6$ is a constant which was calibrated in the simulations such that a secondary educated father teaches his offspring up to secondary level or more in three-fourths of the simulation trials as is empirically reported in the studies on Indian (Azam & Bhatt (2015)) & Latin American (Behrman et al. (2001)) data.

Therefore, $u(\lambda_{max}, b_{max}) = pen.\Big(H_{s,t+1}(\lambda_{max}, b_{max}) - H_{s,t}\Big)$ and $u(0,0) = -\delta H_{s,t}.pen$. The function u is increasing in both inputs.

Father's wage

In each period the labour market is subjected to a labour market binary-shock e_t which can assume two states $e_t = \{e_{low}, e_{high}\}$ with $\mathbf{E}[e_t] = 1$. At any time t, the realized wage of a father with human capital level H_{dad} and labour market shock e_t (with $\bar{G}$ as the exogenous rate of growth of wages) is :

$$WAGE(H_{dad}, t, e_t) = H_{dad} e_t \bar{G}^{t-1} \tag{4}$$

Hence an illiterate person with a basic level of human capital[3] ($\bar{H}$) would earn an expected wage equal to one at time period $t = 1$, which is roughly equivalent to $2/day in the developing world.

Financial assets

The set-up has two financial assets: a riskless single period bond with gross return R_0 and a risky single period stock. The gross return R_{γ_t} of the stock is subjected to a binary-shock γ_t in each period where $\gamma_t = \{\gamma_{low}, \gamma_{high}\}$ can take two discrete states such that $\mathbf{E}[\gamma_t] = 0$. The excess return of stock is denoted by μ and labour market shocks e_t and γ_t may be correlated.

$$R_{\gamma_t} = R_0 + \gamma_t + \mu \tag{5}$$

[3] $\bar{H}$ is calibrated to be equal to 1.0

Given the very limited stock market participation of even college educated households in developing countries (as opposed to intermittent participation) the study attempts to capture it by introducing a one-time entry cost F^c for stock market participation[4]. Bond market participation is assumed to be costless. F^c is fixed and equal to zero if the household head has already participated in the stock market at some point in the past, else it is a positive number for first-time entry. An indicator variable I_t captures the state of the household's participation in the stock market. $I_t = 0$ if the household has never participated and I_t stays one forever after the household participates for the first time (re-entry is costless).

This study attempts to solve the stock market participation and portfolio choice problem of only the parent. The son's portfolio choice problem is not considered by assuming that the father runs the optimization with the assumption of basic financial market access (bond) of the independent offspring. This reduces the solution time of the problem, which otherwise is extremely time intensive.

Children's wages

If the father sends the offspring for work, then the family income gets augmented with the child's income. Alternatively, even if the child doesn't go out for market work, the son may be engaged in home production or farm work which saves the household resources which would otherwise have to be purchased from the market. ILO (2007) survey report on industries in India, Philippines, Ghana, and Uganda documents that industries generally pay children about 20% to 66% of the minimum adult wage.

If a child with human capital $H_{s,t}$ spends λ_t fraction of his time studying ($1-\lambda_t$ fraction working) then the child labour income earned by the family at time t is:

$$CLWAGE(H_{s,t},t) = H_{relv}(1-\lambda_t)\bar{G}^{t-1} \tag{6}$$

Table 1 defines the applicable child-wage rate (H_{relv}) which is a function of age. H_{matric} and $\bar{H}$ are the secondary and base levels of human capital. From $16 \le age \le 20$, it is assumed that 90% of the secondary wage rate applies (twice the minimum adult wage rate) for a child who has completed secondary level of education, else 90% of minimum wage rate (for adults) applies. To reduce one state variable for ease of computation, child-labour wage rates, which are a fraction of the adult minimum wage rate, are assumed to be non-stochastic.

Table 1: Child-labour wage rates

	$0 \le age(son) \le 5$	$6 \le age(son) \le 10$	$11 \le age(son) \le 15$	$16 \le age(son) \le 20$
H_{relv}	0	$0.33\bar{H}$	$0.5\bar{H}$	$H_{relv} = 0.9H_{matric}$ if $H_{s,t} > H_{matric}$ else $H_{relv} = 0.9\bar{H} = 0.9$

[4] intermittent participation suggests the role of per-period transaction costs. However complete non-participation means a high entry cost which acts as a barrier to even one time participation. See Alan (2006)

Expected Utility Maximization
Let $\Omega_t = (e_t, \gamma_t)$ stand for the vector of labour market shock e_t & stock return shock γ_t at any time-period t where e_t and γ_t are individually not correlated over time. They may, however, be correlated cross-sectionally. Let $\Omega^{(t)} = \Omega_1, \Omega_2, \Omega_3 .... \Omega_t$ stand for the history of joint shocks[5] till time t.

Then at the starting time period $t = 1$, and aggregate state Ω_1, with cash-in-hand $X_{f,1}$, education level of the son $H_{s,1}$ and stock market participation state $I_{f,1}$, the father maximizes the expected utility from life-time consumptions (his & offspring's) while also taking into account the utility (per-period) from human capital inputs invested in the son's human capital in the education periods. Therefore, the father's maximization problem is :

$$\max_{C_{f,t}, C_{s,t}, \lambda_t, \alpha_t, b_t} \Bigg(\Big(u(\lambda_1, b_1) \Big) + u(C_{f,1}) + \rho u(C_{s,1}) + \mathbf{E}\Big[\sum_{t=2}^{20} \beta^{t-1} u(\lambda_t(\Omega^{(t)}), b_t(\Omega^{(t)})) \Big] + \mathbf{E}\Big[\sum_{t=2}^{50} \beta^{t-1} u(C_{f,t}(\Omega^{(t)})) + \rho \Big(\sum_{t=2}^{70} \beta^{t-1} u(C_{s,t}(\Omega^{(t)})) \Big) \Big] \Bigg)$$

The father's decision problem is to determine his fifty periods' consumptions ($C_{f,t}$'s), seventy periods' son's consumptions ($C_{s,t}$'s), and twenty joint-periods' optimal education time λ_t's and books' investments b_t's (in terms of foregone consumption). The father also determines the point of entry into the stock market (with one-time entry cost F^c) and the optimal percentage of savings, to be parked in stock (α_t's) for all periods post his entry into the stock market.

Consumption utility is modelled as CRRA, $u(C) = \frac{C^{1-\xi}}{(1-\xi)}$ and the parent's(father) utility from human capital inputs (edu-time λ_t and books b_t) at time t is modelled as $u(\lambda_t, b_t) = pen.\Big(H_{s,t+1}(\lambda_t, b_t) - H_{s,t}\Big)$. Symbols ρ and β represent downward altruism and time-discounting parameters, respectively. A brief discussion of the per-period constraints in the model is presented below:

Current period cash-in-hand balance of father/adult son

$C_{f,t} + C_{s,t} + a'_t + F^c.I_{enter} = X_{f,t} + \bar{G}^{t-1} H_{relv}(1-\lambda_t) - s.b_t$ for $1 \le t \le 20$, $I_{enter} = 1$ if entering stock market for the first time. Otherwise zero.

$C_{f,t} + a'_t + F^c.I_{enter} = X_{f,t}$ independent-father for time-period t: $21 \le t \le 50$

$C_{s,t} + a'_{st} = X_{s,t}$ adult-offspring for time period t: $21 \le t \le 70$

$X_{f,t}$ represents the total real resources in hand with the father and it includes the returns from last period's investments in assets as well as current period labour income. The LHS of the first equality represents consumption expenditures, stock market entry cost if applicable and savings while the RHS represents

[5] $\Omega^{(t)}$ is separate from Ω_t (which signifies the realized time period t joint shock). For t=2, $\Omega^{(t)}$ will be $\Omega^{(2)} = \Big((\gamma_{low}, e_{high}), (\gamma_{high}, e_{high}) \Big)$

total resources in hand including child wage earning ($\bar{G}^{t-1}H_{relv}(1-\lambda_t)$) and expenses in books' ($s.b_t$) at time t. $(1-s)$ represents the discount fraction in books. a'_t and a'_{st} represent the father and the adult offspring's end of period savings at time t respectively.

Next period cash-in-hand balance of father/adult son

$a'_t R_{p,t+1} + \bar{G}^t e_{t+1} H_{dad} = X_{f,t+1}(\Omega_{t+1})$ for working father for t: $1 \le t \le 39$ *where* $\Big(R_{p,t+1} = R_0 + \alpha_t(R_{\gamma_{t+1}} - R_0)\Big)$ where $\alpha_t = 0$ if the household hasn't entered the stock market by paying the one-time entry cost F^c. Else $R_{p,t+1}$ is the portfolio return from stock and bond investments.

$a'_t R_{p,t+1} = X_{f,t+1}(\Omega_{t+1})$ for retired-father for t: $t \ge 40$

$a'_{st} R_0 + \bar{G}^t e_{t+1} H_{s,t} = X_{s,t+1}(\Omega_{t+1})$ adult-son for time period t: $21 \le t \le 59$ (the son is assumed to have only bond access)

$a'_{st} R_0 = X_{s,t+1}(\Omega_{t+1})$ for retired-son for t: $t \ge 60$

The state contingent resources at $t+1$ for aggregate state Ω_{t+1} is represented by $X_{f,t+1}(\Omega_{t+1})$ which is equal to the returns ($R_{p,t+1}$) from the last year's savings (a'_t) added to the father's $t+1$ period wage income ($\bar{G}^t e_{t+1} H_{dad}$).

Offspring's human capital

$H_{s,t+1} = f(\theta, \lambda_t, H_{s,t}, b_t) = \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} + H_{s,t}(1-\delta)$

The child's next time-period human capital depends on the amount of investments in education time and books as well as the human capital level in the previous time period.

Borrowing constraint

$a'_{st} \ge 0$ & $a'_t \ge 0$ for all relevant adult offspring and father's time periods.

Father/adult-son's savings cannot be negative.

Non negative constraints

$0 \le \alpha_t \le 1$ for time period $1 \le t \le 50$

$0 \le \lambda_t \le 1$ for time period $1 \le t \le 20$

Fraction of savings in stock α_t and fraction of time invested in school, λ_t, should be between zero and one (including both values).

Upper cap on books

$b_t \le b_{max}$ for $1 \le t \le 20$

b_{max} is the upper cap in books' investment in any time period.

The human capital accumulated by the son in the first twenty periods determines his level of consumption (and utility from consumption) in the adult stage. The father cares about the son's utility from consumption through the downwards altruism parameter ρ. Beyond that, the father also explicitly values the human capital acquisition of the son for human capital's sake, and the model tries to capture it using the utility from human capital inputs term $u(\lambda_t, b_t)$. The father solves the multi-period problem at the beginning under the assumption that the level of human capital he leaves his adult son with, will determine his income and consumption level as an independent adult. The model doesn't include future generations.

First order conditions and solution methodology

The multi-period optimization problem is solved employing backward induction where the strategy is to address the last period first. The value in any period t can be recursively written in-terms of optimum variables of that period and the next period value.

For any time $1 \leq t \leq 19$, and joint state of the world $\Omega_t = (e_t, \gamma_t)$, if $V^{(t)}(X_{f,t}, H_{s,t}, I_{f,t})$ is the value for holding $X_{f,t}$ resources-in-hand, $H_{s,t}$ amount of human capital of the offspring and stock market participation state $I_{f,t}$ then, the maximization problem can be recursively written as:

$$
\begin{aligned}
V^{(t)}(X_{f,t}, H_{s,t}, I_{f,t}) = \max_{C_{f,t}, C_{s,t}, \lambda_t, \alpha_t, b_t} & u(\lambda_t, b_t) + u(C_{f,t}) + \rho u(C_{s,t}) + \\
\sum_{\Omega_{t+1}} \pi(\Omega_{t+1}) \beta V^{(t+1)} & \big(X_{f,t+1}(\Omega_{t+1}), H_{s,t+1}(\Omega_{t+1}, I_{f,t+1}(\Omega_{t+1}))\big)
\end{aligned}
\tag{7}
$$

At any time period t, a household without stock market access has to decide whether to enter the stock market by paying the first-time entry cost F^c. The household chooses that by comparing the discounted future expected values from participation and non-participation.

$$
\begin{aligned}
V^{(t)}(X_{f,t}, H_{s,t}, I_{f,t} = 0) = \max_{I_{enter}=0,1} \Bigg\{ & \Big(u(\lambda_t, b_t) + u(C_{f,t}) + \rho u(C_{s,t}) + \sum_{\Omega_{t+1}} \pi(\Omega_{t+1}) \beta V^{(t+1)}(X_{f,t+1}, H_{s,t+1}, I_{f,t+1} = 0) \Big), \\
& \Big(u(\lambda_t, b_t) + u(C_{f,t}) + \rho u(C_{s,t}) + \sum_{\Omega_{t+1}} \pi(\Omega_{t+1}) \beta V^{(t+1)}(X_{f,t+1}, H_{s,t+1}, I_{f,t+1} = 1) \Big) \Bigg\}
\end{aligned}
\tag{8}
$$

2.1. Other FOCs for $1 \leq t \leq 19$

Apart from the stock market participation decision, at any chosen time period in the joint-phase, the optimum would be the balance of five different forces: dad and son's utility from consumption; higher resources-in-hand in the next period earned through asset (portfolio) returns and therefore higher next period value; higher human capital level of the offspring (and higher value) due to more time in school (foregoing child labour earnings) or more investments in non-time inputs like books. When none of the inequality constraints bind, the optimum is characterized as the trade-off of these competing forces. The cases when the inequality constraints bind need to be tackled on a situation-by-situation basis.

The father and the offspring's utility from consumption trade-off is presented in equation 9 where ρ is the altruism parameter.

$$
u_c(C_{f,t}) = \rho u_c(C_{s,t}) \tag{9}
$$

One unit of sacrificed consumption by the father leads to $u_c(C_{f,t})$ loss in his present utility (RHS of equation 10). The same unit when invested, leads to $R_{p,t+1}$ returns[6] and $V_1^{t+1}(.)R_{p,t+1}$ higher tomorrow's value[7] due to the additional resources at time $t+1$. At the optimum they should be equal. However, when the borrowing constraint is binding ($\mu_{borr,t}$ positive) the marginal utilities won't be equal and the household consumes all its current resources.

$$\sum_{\Omega_{t+1}} \left(\beta \pi(\Omega_{t+1}) V_1^{(t+1)}(.) R_{p,t+1} \right) + \mu_{bor,t} = u_c(C_{f,t}) \tag{10}$$

An additional unit investment in non-time inputs like books (or alternatively, education-time) will lead to s units[8] (or $\bar{G}^{t-1} H_{relv}$ units for edu-time) lower father's consumption due to incurred cost on books (or foregone child labour earnings), leading to $s.u_c(C_{f,t})$ (or $\bar{G}^{t-1} H_{relv} u_c(C_{f,t})$) fall in utility (RHS of equations 11 & 12). However that extra unit of books invested (or education-time) in human capital production increases the offspring's human capital next-period by $f_4(.)$ units[9] (or $f_2(.)$ for edu-time) and increases next-period expected value by $f_4(.)V_2^{t+1}(.)$ (or $f_2(.)V_2^{t+1}(.)$). Additionally, due to $f_4(.)$ units of additional human capital production, the father realises $pen.f_4(.)$ amounts of extra utility owing to his valuing of his son's human capital through the $u(\lambda_t, b_t)$ term[10]. The constant denoted by $pen(= 1.6)$ represents the increase in father's utility owing to an extra unit of human capital production of the offspring. If the books' upper limit constraint ($\mu_{b,t}$) is non-binding, the fall in utility ($s.u_c(C_{f,t})$) should equate the expected gain ($f_4(.)V_2^{t+1}(.) + pen.f_4(.)$) at the optimum. If the constraint binds then the books' investment will be a corner point.

$$f_4(.) \sum_{\Omega_{t+1}} \left(\beta \pi(\Omega_{t+1}) V_2^{(t+1)}(.) \right) + pen.f_4(.) - \mu_{b,t} = s.u_c(C_{f,t}) \tag{11}$$

$$f_2(.) \sum_{\Omega_{t+1}} \left(\beta \pi(\Omega_{t+1}) V_2^{(t+1)}(.) \right) + pen.f_2(.) + \mu_{\lambda_m,t} - \mu_{\lambda_x,t} = \bar{G}^{t-1} H_{relv} u_c(C_{f,t}) \tag{12}$$

If the household has stock market access (incurred the entry cost once) then the optimal portfolio choice (equation 13) is characterised as such that when there is a unit increase in the fraction of stock (α_t) in the savings, and it leads to ($R_{\gamma_{t+1}} - R_0$) increase in expected returns, there would not be any change in the expected future value. In the event of any such possibility α_t would not be the optimum.

$$\mu_{\alpha_m,t} - \mu_{\alpha_x,t} + \sum_{\Omega_{t+1}} \left(\beta \pi(\Omega_{t+1}) V_1^{(t+1)}(.) a_t'(R_{\gamma_{t+1}} - R_0) \right) = 0 \tag{13}$$

[6] if stock market non-participant then $R_{p,t+1} = R_0$

[7] $V_1^{t+1}(.)$ refers to the partial derivative of $V^{t+1}()$ w.r.t first argument which is cash-in-hand

[8] (1-s) is the fractional discount on books

[9] where $f()$ is the human capital production function and $f_4()$ is the partial derivative w.r.t books

[10] $u(\lambda_t, b_t) = pen.\left(H_{s,t+1}(\lambda_t, b_t) - H_{s,t} \right)$ where $H_{s,t+1}(\lambda_t, b_t) = f(\theta, \lambda_t, H_{s,t}, b_t)$; $\frac{\partial u(\lambda_t, b_t)}{\partial b_t} = pen.f_4(.)$

Similarly, other time periods can be solved[11].

3. Benchmark parameter values

3.1. The three representative income levels

The study numerically computes the impact of financial market (bond & stock) participation on a household's human capital investments and consumption expenditures. In the presence of fixed one-time entry costs for stock market participation, the study computes the optimal portfolio choice graphs for three household heads with different levels of incomes and human capital investment options in their offsprings. The sensitivity of the portfolio choice graphs for different risk and return parameters are also evaluated.

A linear relation between parental human capital and the income level of the parent is assumed in this study. Human capital levels of the fathers have been calibrated as per empirically reported income ratios of different education categories. The literature on developing countries reports a 4.5 times higher wage for college educated workers as compared to illiterate workers. Similarly, secondary educated workers earn on average twice the income of illiterate people (Agrawal (2012)). Therefore the study calibrates the human capital level of the college and secondary educated fathers as five and two times that of an illiterate father. Sensitivity analysis of portfolio choice results with incomes is reported (in section 6) to make the results generalizable.

To reduce one state variable for ease of computation, the model considers only transitory income shocks for analysis. Incorporating permanent shocks would additionally capture the effect of shock persistence over time but at the cost of an additional state variable. Since the focus of this study is to evaluate the difference in portfolio allocation of a parent as compared to that of a single adult, the study opts for the simpler set-up by excluding permanent shocks.

The study considers three fathers as proxies for three human-capital levels of the parents: The College Dad, The Secondary Dad, and The Illiterate Dad. Apart from income and education levels, the households are assumed to be heterogeneous with regard to their human-capital investment options. The summary of the three dads is included in Table 2.

Studies like Rhine & Greene (2013), Pal & Pal (2012), Djankov et al. (2008) clearly highlight the low financial access levels of poor, uneducated households in developing countries. This study evaluates the effect of bond market and stock market participation on consumption and educational investments of the three representative households. Other than financial exclusion, access to school and health resources are also limited for poor-illiterate households due to discrimination, distance, and other supply bottlenecks.

[11] see also Thakurata & D'Souza (2018)

The study attempts to capture this by setting different upper-caps on non-time educational investments (like books) for the different income categories.

For developing countries like India, Azam & Kingdon (2013) report that educational expenditures on school fees, books, private tutors, transport and others constitute around 4.3% of a household's expenditure. The cost recovery levels in public education in Asia are reported to be around 1.7%, 16% & 11.5% for primary, secondary, and higher education, respectively (Psacharopoulos (1988)). This study, therefore, assumes that a household bears only 50% of the actual expenditures on non-time educational inputs (like books) and the upper-cap on these investments is set at 25% of household income[12]. Only the college dad's upper-cap on books has been fixed at 20% of the father's income. The sensitivity of the results to these parametric assumptions is presented in Section 5.

Table 2: The three benchmark households

	H_{dad}	Books'-Discount $(1-s)$	Books-Max (b_{max})
Illiterate Dad	1.0	50%	0.25
Secondary Dad	2.0	50%	0.5
College Dad	5.0	50%	1.0

*3.2. **Consumption and human capital parameters***

Other than the parameter *pen* which is explicitly calibrated in this study, the other parameters have been used from literature.

Calibration of *pen* : Studies like Woessmann (2004), Black et al. (2005) have documented the high correlations between parental human capital and the offspring's educational attainment. Parameter *pen*, which captures the father's per-period utility from a unit increase of son's human-capital, has been calibrated such that over ten thousand simulation trials, a secondary educated father ($H_{dad} = 2$) educates his offspring till secondary level or more for three-fourths of the trials and an illiterate parent with zero years of education never sends his/her child to school. Studies like Azam & Bhatt (2015) & Behrman et al. (2001) on Indian and Latin American data have reported these empirical observations and after calibration, the parameter *pen* was set as equal to 1.6.

The financial and human-capital parameters used in this study are presented in Tables 3 & 4. The risk aversion coefficient has been assumed at $\xi = 5$. Human-capital production function parameters (*aa*, *bb*, *cc*), depreciation (δ) & ability (θ) are similar in magnitude to the values mentioned in the studies cited in Browning et al. (1999)[13] with some adjustments to generate average returns of 11% when the offspring

[12] assuming 4.3% expenditure is divided evenly between private and public expenditures with 0% and 10% cost recovery, the % discount amounts to 45%. Also the total investments amount to 21.5% + 2.15% = 24% of expenditure (approx) or 18% of income (assuming 25% household savings as reported in surveys like Shukla (2010)). Therefore discount % is set at 50% and the upper-cap at 25% of income.

[13] see also Ludwig et al. (2012), Manuelli & Seshadri (2014)

is engaged full time in school education with the upper limit of available books ($b_{max} = 1$). Downward altruism coefficient of the father (ρ) has been assumed to be equal to one in the joint periods ($1 \leq t \leq 20$) and 0.7 for the adult son's life-cycle. Since the study refrains from explicitly modelling the health-related expenditures of the offspring, it assumes equal father-son consumption expenditures in the joint periods ($\rho = 1$) to partly make up for that even though the son might consume lesser than the father.

Table 3: Benchmark : human capital & consumption parameters

ξ	θ	bb(=aa)	cc	δ	ρ	pen	$\bar{H}$	H_{matric}
5.00	0.160	0.750	0.20	1.80%	1, 0.7	1.6	1	2

3.3. ***Financial parameters***

The financial parameters are similar in magnitude to the ones reported in Mehra (2006) for India. The bond rate has been set at $r_0 = 2\%$[14], and the expected stock return has been assumed as $\mathbf{E}[r_s] = 13\%$[15]. The variance of shocks to stock returns has been assumed as $var(\gamma_t) = 0.1369$. The real return of a financially excluded household has been assumed as $r_{nofin} = -5\%$ which is the average level of inflation in emerging economies as reported in ILO (2018). A study which measures the magnitude of income shocks in Indian villages (Jacoby & Skoufias (1997)) reports the variances of idiosyncratic shocks as 0.048, 0.095 and 0.241 for three sampled villages. Drawing from the reported values, this study assumes the variance of the logarithm of transitory income shocks[16], $var(log(e_t))$, at 0.095. Income and stock return shocks have been assumed to be mutually uncorrelated ($corr = 0$). The sensitivity of the results for correlated shocks gets separately discussed as part of sensitivity analysis. Time-discount factor is assumed as $\beta = 0.98$ and $\bar{G} = 1\%$ is the exogenous growth rate of income, which has been benchmarked against the global wage growth rate (ILO (2018)).

Table 4: Benchmark : financial parameters

R_0	$\mathbf{E}R_s$	r_{nofin}	var(γ_t)	var($log(e_t)$)	corr	β	$\bar{G}$
1.02	1.13	−5%	0.1369	0.095	0	0.98	1%

4. Financial widening outcomes : Three benchmark households

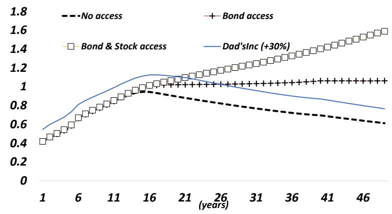
This section presents the simulated profiles of the three representative households for different financial access levels of the household heads. Calibration of the parameter (pen) capturing the parent's affinity for the offspring's human-capital was done such that the secondary father educates his offspring till secondary or more in three-fourths of the simulation trials and the illiterate dad's offspring never goes to school. Repeated simulation trials were run by drawing realizations of stock return and income shocks to generate average consumption and asset profiles of the three households. The life-cycle profiles reported here are the cross-section means of 10,000 trials. The offspring's value function has been computed by assuming

[14] $R_0 = 1.02$

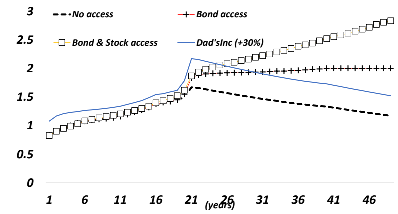
[15] $\mathbf{E}[R_s] = 1.13$ with an equity premium of 11%

[16] where $E[log(e_t)] = 0$

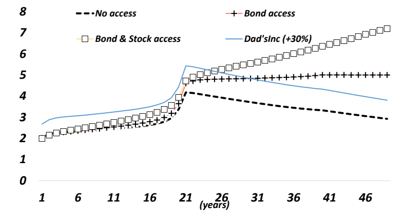
bond market access[17] for the offspring to ensure that differences in the offspring's values occur only out of differences in educational outcomes of the offspring due to differential parental investments (owing to heterogeneity in parental incomes and parental financial access levels). This assumption significantly lowers down the computation time, which runs into weeks.



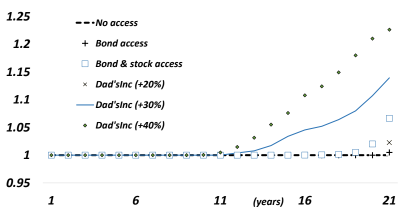
(a) Illiterate dad: consumption



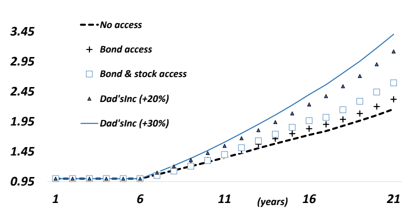
(b) Secondary dad: consumption



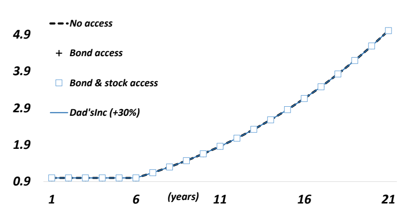
(c) College dad: consumption



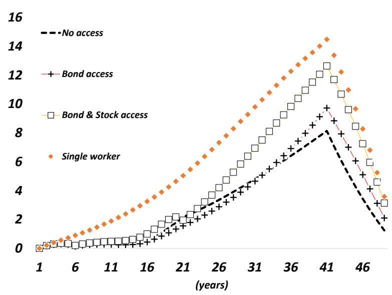
(d) Illiterate dad: human capital



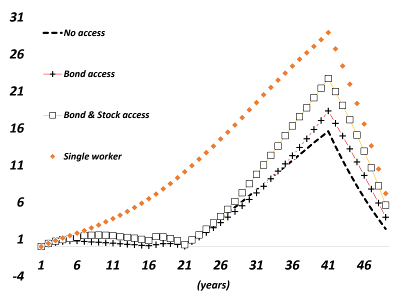
(e) Secondary dad: human capital



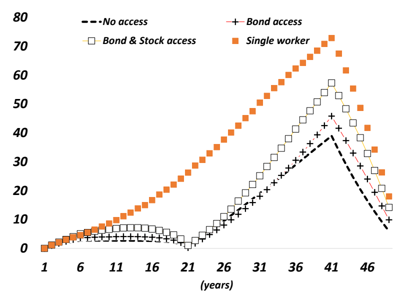
(f) College dad: human capital



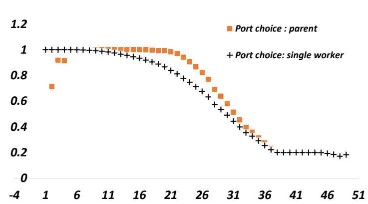
(g) Illiterate dad: assets



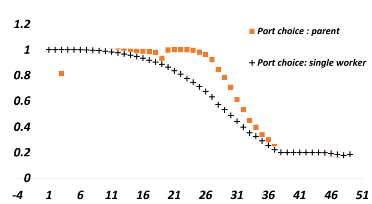
(h) Secondary dad: assets



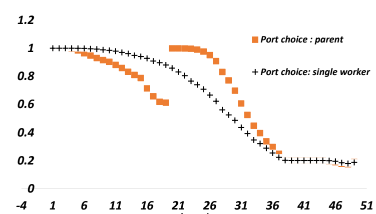
(i) College dad: assets



(j) Illiterate dad: portfolio choice



(k) Secondary dad: portfolio choice



(l) College dad: portfolio choice

Figure 2: Illiterate, secondary & college dads' per-capita consumptions, human capital outcomes of the offsprings, household assets, & portfolio choice graphs for twenty joint-periods & thirty independent & retired periods.

Figure 2 plots the educational outcomes of the offsprings, the consumptions (per-capita) and the asset profiles of the three households while Table 5 presents the corresponding values of the variables for the

[17] while optimizing, the parent makes the assumption that after 20 years the adult offspring is unlikely to be financially excluded due to the rapid spread of banking access in developing countries. Portfolio access of the offspring is ignored to save computation time.

Table 5: Human capital outcomes & consumptions of three representative households for different financial access levels & incomes of household heads. All % changes for bond & portfolio access and higher incomes are over the no-access (N) values.

	Illiterate Dad				Secondary Dad				College Dad			
	N	**B**	**P**	**Δ Inc 20%,30%**	**N**	**B**	**P**	**Δ Inc 20%,30%**	**N**	**B**	**P**	**Δ Inc 30%**
HC	1	0%	0%	0%,14%	2.16	7.5%	20.2%	44.4%, 57.7%	5	0%	0%	0%
C_j	0.76	4.2%	5.7%	15.8%, 21.4%	1.18	0.1%	2.2%	10% , 16.4%	2.5	3.7%	11.3%	32.4%
C_{id}	0.78	33.1%	59.8%	17.7%, 24.8%	1.49	30.4%	48.7%	19.9% ,29.9%	3.71	30.3%	50.2%	30.1%
C_{rd}	0.64	65.1%	135.9%	17.9%, 25.2%	1.23	62.5%	119.5%	19.9% , 29.9%	3.08	62.4%	122.9%	30.1%
$\sum PV(C_{j,t})$	12.3	3.8%	5.2%	16%, 21.6%	19.3	0%	2%	10.3% , 16.8%	41.14	3.3%	10.4%	32.3%
$\sum PV(Inc_{j,t})$	18.2	0%	0%	20%, 30%	36.4	0%	0%	20% ,30%	91.1	0%	0%	30%

N: no access; B: only bond access; P: bond & stock access; Δ Inc 20%: when dad's income is 20% higher with no financial access
HC: human capital of the offspring; C_j: average per capita consumption of dad/son in the twenty joint periods; C_{id}: independent dad's average per-capita consumption; C_{rd}: retired dad's average per-capita consumption; $\sum PV(C_{j,t})$: present value of per-capita consumptions in the twenty joint periods; $\sum PV(Inc_{j,t})$: present value of father's incomes in the twenty joint periods.

base case when the fathers are financially excluded and the percentage changes in the variables for higher levels of financial access and incomes of the three fathers. Figure 2 also additionally plots the optimal portfolio choice graphs (in the absence of any stock market entry costs) for the three fathers when they are assumed to participate in the bond as well as the stock market.

Illiterate household: For the illiterate dad household, results show that a working child is an optimal outcome of the life-cycle model. The son is never sent to school, and the total income (including the child's earnings) is almost completely spent on consumption (the assets in the joint-periods are very low). Overall, it is an inter-generational poverty trap (self-perpetuating) where the child's future gets traded-off and the son, like father, stays at the minimum level of human capital ($\bar{H} = 1$) with no schooling (Table 5: row 1, col 1). Financial widening to bond and stock markets are revealed not to have any impact on human capital, though access to bond and bond as well as stock markets do help in raising the consumption levels in the joint-periods by 4.2% & 5.7% respectively which is crucial for the child's nutritional development. The independent & retired dad's consumption levels rise significantly (59.8% & 135.9% increase for full financial access) owing to his higher assets in the later periods of his life.
For this income category, a basic income support of 30% or higher to the household head, remains the only way of inducing this category to invest in education as opposed to child labour (Table 5 row 1, col 5). Figure 3 plots the simulation results of the son's human capital outcomes for a range of father's incomes with and without bond market access. If the father is a bond market participant, then it takes an additional income support of 88% of the minimum wage to push the household out of the intergenerational poverty trap where the next generation's human capital (and income), on average, will exceed the previous generation's income (and human capital). However, when the father doesn't participate in the bond market, the household needs an income support of 100% of the minimum wage for the offspring to reach the same

human capital level of 1.88[18]. Therefore a combination of income support and banking access is more efficient in pushing this category of the household out of the intergenerational poverty trap[19].

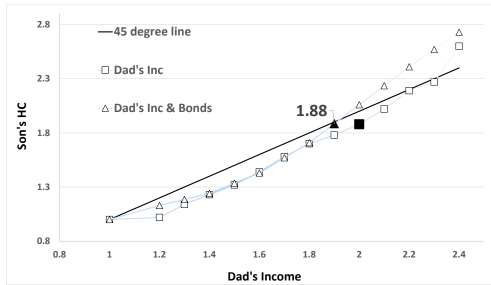


Figure 3: Sensitivity of son's human capital outcomes with increases in dad's income and bond access

Secondary household: The simulation results reveal that the secondary dad's offspring gets partial education and on average ends up with an 8% higher human-capital level than the father for the household which doesn't participate in the bond or stock market. Access to basic financial savings instruments like bonds, raises the human capital outcome of the offspring by an additional 7.5% whereas having a savings portfolio comprising of bonds, as well as stocks, improves it by a further 12.7% (Table 5). For this household category, the higher return on savings due to market access gets mostly invested in human-capital rather than in consumption (Figures 2 (b), (e)). As a result, there is an only a minor change in joint-periods' consumption values (only a 2.2% increase for full financial access). Higher financial access impacts the independent & retired dad's consumptions significantly (48.7% & 119.9% increases for complete financial access) due to non-existence of his son's human capital and consumption expenditures in this phase of his life.

Table 5 also presents the % changes in variables for income enhancements of 20% & 30% of the household head. A 20% income support to the secondary educated father raises the human capital outcome of the son by 44.4% (income elasticity 2.2). The present value of additional resources required for the extra income support in the joint-periods[20] is 36.4*0.2 = 7.28 (Table 5, row 6, col 5). Therefore the cost of raising human capital by 100% is 14.56 units in present value terms. Additional stock market participation by a household with only bond market access is seen to raise the human capital of the offspring by an additional 12.7%. The simulation trials in the next section estimate the entry cost for stock

[18]in all scenarios the adult offspring is assumed to have bond market access to ensure that differences in the offspring's values arise only due to differences in parental investments in the son's human capital owing to income or financial access heterogeneity of the parents. Also after 20 years, financial inclusion levels would be universal so the son won't be financially excluded. The figures quoted here are upper bounds of the required income supports as the father doesn't take into account the dynastic framework for optimization. Presence of future generations would increase father's utility from son's human capital so the external support required would be lower. The study ignores the grand child to reduce computation time

[19]for the prevailing level of stock market entry costs in developing countries like India, this household is seen to be a stock market non-participant. The next section analyses participation rates & entry costs

[20]the asset profile of this household revels that the dad doesn't carry any asset into the independent period. Therefore any incremental income in the joint-periods gets completely spent on consumption & human capital investments in the joint-periods.

market participation at around 155% of the minimum wage. Assuming that 1.55 units[21] of resources are spent by the policy maker to enable this category of the household to enter the stock market, then the human capital improvement will be an additional 12.7%. In terms of resource efficiency, stock market participation, through its impact on return on savings, is 33% more effective in improving human capital than income support. This is, at best, only a rough estimate as the study considers improvements from return on assets as the only criterion for the trade-off. It makes the simplifying assumption that economic benefits from participation may induce households to overcome the structural reasons for non-participation like financial illiteracy and lack of information regarding stock markets.

College household: The son of a college educated father gets complete schooling with investments in education-time & books at the upper limit at all time periods (Table 5, row 1, col 9). Only higher ability or higher access to educational inputs (higher b_{max}) can increase the human-capital level of the offspring any further. This high-income category's educational investments are completely inelastic to financial access levels of the household. Participation in the bond market raises the joint-periods' consumption by 3.7% whereas participation in bond, as well as the stock market, raises it by an additional 7.6%. The impact of financial market participation is higher on the independent & retired dad due to his higher asset levels.

Asset profiles of the three households: Figures 2(g)-(i) plot the asset profiles of the three fathers for different levels of financial market participation of the fathers. For comparison, the figures also plot the asset profiles of single independent adult workers with bond & stock market access and the same incomes as the three household heads. In contrast to the asset profiles of the three single adult workers, strict borrowing constraints lead to double humped asset profiles of the three fathers with assets in the first twenty periods much lower than the independent fathers' asset levels. The fathers would have ideally wanted to borrow from the future time periods to meet the expenses of their higher educational and consumption requirements in the joint periods. However, binding borrowing constraints lead to the fathers running two distinct optimizations. The low asset levels in the joint-phase is a reason for the households not being able to take strong advantage of the higher return on savings made available through bond and stock market access.

5. Optimal portfolio choice, entry costs & stock market participation

Two contributions of this work are the estimation of the fixed entry costs for stock market participation in developing countries as well as the numerical computation of the optimal portfolio choice graph of a parent as different from that of a single adult worker. This study specifically investigates if the percentage of stocks in the savings portfolio of a parent, apart from varying with age and wealth, also varies with the stage of his life in the intergenerational life-cycle.

[21] Entry costs may differ across income categories if measured in opportunity cost of time. However, a common entry cost of 155% best explains the observed non-participation levels captured in the surveys.

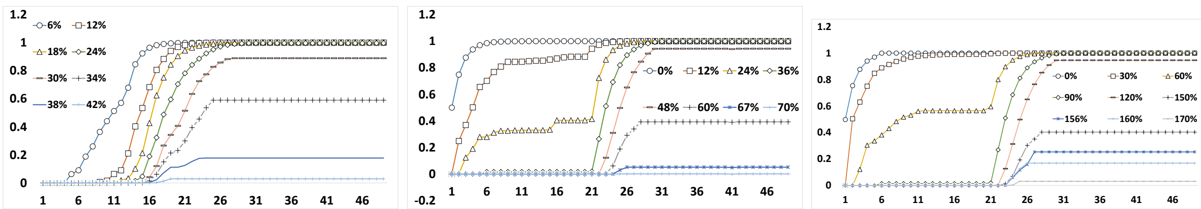
Portfolio choice without entry costs: Figures 2(j)-(l) plot the life-cycle portfolio allocations of the three fathers (with assumed stock market participation) from $t = 1$ to 50 when labor market shocks are uncorrelated with stock market shocks, and there are no entry costs for stock market participation. The fathers' life-cycles can be divided into three sub-periods: joint-phase, independent phase, and retired phase. For uncorrelated shocks, labour income mimics a riskless asset and the portfolio rule follows a pattern where stocks are preferred in the early periods due to the endowment of riskless labour income. The ratio of the present value of future labour income to wealth is an important variable in portfolio choice. In the joint-periods, the present value of future labour income is high and wealth remains consistently low (compared to a single worker's wealth profile) due to binding borrowing constraints and higher expenditures on consumption and human capital. Hence due to very low asset levels in the joint periods, portfolio choice is mostly stock for a significant part of the parent's working life as compared to a single adult worker whose optimal portfolio graph monotonically tilts from stock to bond with time (Figures 2 (g)-(l)). After that, the wealth increases in mid-life as the parent is saving for retirement, and due to lower earning periods left, the present value of future labour income starts to fall. Hence, the portfolio rule starts shifting towards bond a few years before retirement. Post-retirement, the portfolio allocation remains constant for all the three fathers due to the absence of future labour income streams. Compared to a single worker's optimal portfolio allocation, a parent's portfolio choice graph remains loaded towards stock for a longer time-frame as his assets remain low throughout the joint-periods due to his higher expenses on the offspring. The college father's curiously large portfolio tilt towards bonds in the joint periods gets discussed in the sensitivity analysis.

*5.1. **Stock market entry costs & participation rates***

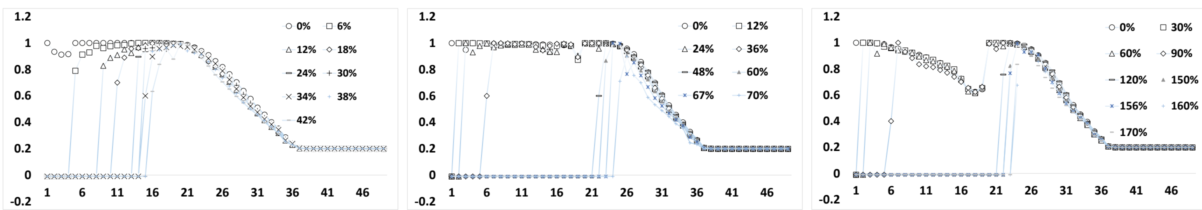
Figure 4 plots the participation rates and optimal portfolio choice graphs of the three households for different levels of stock market entry costs. For entry costs higher than 42% of the minimum adult wage, the illiterate dad never enters the stock market. For costs lower than that, the entry time in the stock market gets delayed with rising entry costs as the asset levels are lower in the earlier periods. The portfolio choice graphs for different levels of entry costs (including zero costs) mimic each other except for the first period of participation when the tilt towards bond is greater due to the incurred fixed costs of entry.

The secondary and graduate dads' portfolio choice graphs follow the same trend (as in the absence of entry costs) including the tilt towards bonds in the period of entry into the stock market. However, one crucial difference is that both these households enter the stock market either in the first five time periods when the son's education is yet to start or after the adult son has separated. In the presence of positive entry costs for stock market participation, both these households seem to display no appetite for stock market entry when human capital investments in their offsprings are in progress.

At entry cost levels of 35% of their annual incomes (70% & 175% of the minimum wage respectively) the two fathers never enter the stock market. A SEBI (2015) investor survey on investors in India reported 25%, 5% & 0% stock market participation levels for households with college level, secondary level and no education of the household heads. The simulation trials of this study mimic the reported data for an entry cost of 155% of the minimum adult wage (or 31% of the graduate educated father's annual wage). At this level of entry cost, the study finds a 25% participation of the college dad and 0% participation of the secondary and illiterate dads. Therefore, if the study has to peg a single one-time entry cost for all income categories of Indian households, then the cost estimated by this study is approximately 155% of the minimum adult wage or around 1100 dollars[22].



(a) Illiterate dad: participation rates vs entry costs (b) Secondary dad: participation rates vs entry costs (c) College dad: participation rates vs entry costs



(d) Illiterate dad: portfolio choice vs entry costs (e) Secondary dad: portfolio choice vs entry costs (f) College dad: portfolio choice vs entry costs

Figure 4: Stock market entry costs: sensitivity of participation rates & portfolio choice graphs of three households for different entry costs. Entry costs are measured as % of minimum wage.

5.2. *Sensitivity of portfolio choice graph to higher risk parameters*

Studies like Claessens (2006) document the low access levels of households to even basic financial services in developing countries. Apart from income and wealth, education is found to be an important variable which explains the lack of banking access (Rhine & Greene (2013), Pal & Pal (2012), Djankov et al. (2008)). Even though households' participation in the stock markets is very limited in developing countries, this study considers both the secondary and college educated households for exploring the sensitivity of the optimal portfolio choice graph to risk and return parameters.

For college and secondary educated households, Figure 5 presents the households' adjustments in portfolio allocation and books' investment profiles when the risk aversion coefficient, the standard deviation of stock returns, the standard deviation of income shocks and the correlation between labour and stock market shocks are higher than the benchmark values. For comparison, the corresponding portfolio choice graphs for secondary and college educated single adult workers have also been presented[23].

[22]with a minimum wage of around 2 dollars/day

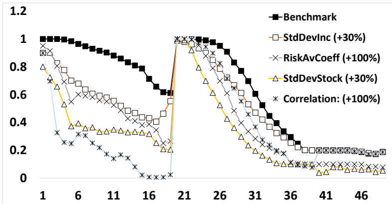
[23]entry costs have been assumed to be zero

Sensitivity of college dad's portfolio : With increasing risk, the college-educated household's portfolio choice graph forms a deep valley (in each of these cases) indicating a greater shift towards bonds as compared to a single worker's portfolio in the first twenty time periods (Figures 5(a) & (c)). However, after the adult son separates at $t = 21$, the now independent dad is seen to immediately shift his investments back to stocks, thus reflecting a higher appetite for stocks. Ceteris-paribus, the risk-taking ability of a household of a single earning member with a dependent offspring is lower than that of a lone individual. In other words, the value function would have a higher curvature for a two-member household vis-a-vis a single member household[24]. Since the weight-age of stocks in the savings portfolio is inversely related to curvature, the portfolio rule witnesses a switch towards bonds in the joint-periods with increasing risk. Post the separation of the son, the father due to his low asset levels as compared to a single worker immediately switches his portfolio back to stock. The fraction of stock in the portfolio gradually falls as the father approaches retirement.

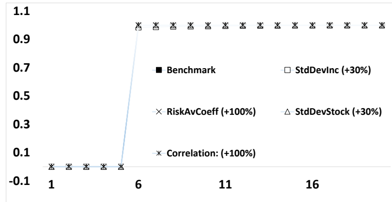
Sensitivity of secondary dad's portfolio: It is interesting to note that the shift towards bonds due to higher risk is lower for the secondary dad (compared to college dad) in all the scenarios (Figure 5(d)). Figure 5 (e) which plots the books' investment profiles of the secondary household, depicts that unlike the college dad (Figure 5(b)), the secondary dad's books' investments are lower than the maximum limit ($b_{max} = 0.5$) in the majority of the joint time periods and hence there exists margin for adjustment. The presence of endogenous human capital investment options (education-time λ_t and books b_t) increases the secondary household's risk appetite and works as a risk mitigating strategy against financial asset risk. In the face of higher risk, the secondary household's portfolio is still predominantly stock as compared to college household due to the option of calibrating the human capital investments based on financial returns. The same option doesn't exist for the college dad as his human capital investments are at the upper limit for the chosen range of risk and stock return parameters (Figure 5(b)).

If human capital investments are elastic then the ability to adjust human capital investments gives the household another margin of adjustment and makes it more tolerant of financial risk. Endogenous human capital investments act as similar to endogenous labour income (Bodie et al. (1992)) where they increase an individual's stock holding appetite and shift the portfolio towards the stock. However, if human capital investments are inelastic like for the college dad, then the presence of a dependent offspring reduces the household's appetite for stocks and therefore shifts the portfolio more towards bonds as compared to single workers.

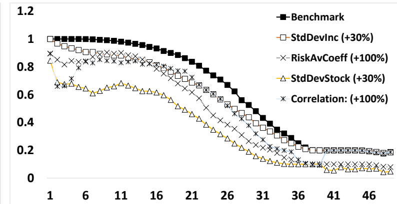
[24]if X is the cash-in-hand with a single member household then the value is u(X). In a two member household, value would be $V(X) =$max $\{u(c_f) + u(c_s)\}$ s.t $c_f + c_s = X$. The optimal value is $V(X) = 2u(X/2)$. Assuming CRRA utility; $u(X) = \frac{X^{1-\theta}}{1-\theta}$ and $2u(X/2) = 2\frac{X/2^{1-\theta}}{1-\theta}$. The effective θ' s.t $2\frac{X/2^{1-\theta}}{1-\theta} = \frac{X^{1-\theta'}}{1-\theta'}$ will be satisfied when $\theta' > \theta$. The fraction of stock in the portfolio is inversely proportional to θ



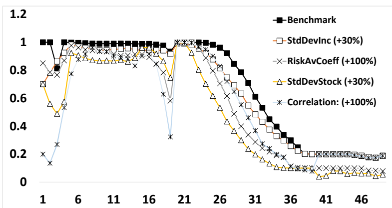
(a) College dad: portfolio choice



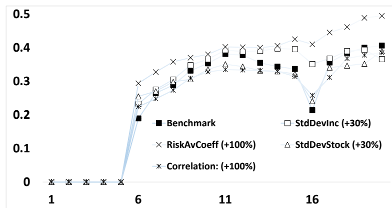
(b) College dad: books' investment



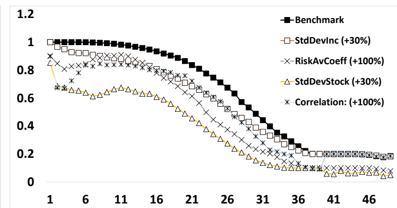
(c) College adult worker: portfolio choice



(d) Secondary dad: portfolio choice



(e) Secondary dad: books' investment



(f) Secondary adult worker: portfolio choice

Figure 5: Sensitivity of college and secondary dads' lifecycle portfolio choice & books' investment graphs for higher risk aversion coefficient, higher standard deviations of stock market shocks & income shocks and high correlation between stock and labour market shocks

6. Sensitivity: affinity for human capital parameter (*pen*) & downward altruism (ρ)

Figure 6 summarizes the books' investments, human capital outcomes & per capita consumption results when the intergenerational parameter ρ (altruism) and the parental valuing of human capital parameter (*pen*) are switched off. The figures reveal that the intergenerational parameters play a significant role in the determination of human capital investments and therefore, joint period consumptions of the secondary and college educated households. Switching off the parameter responsible for the valuing of human capital ($pen = 0$) lowers educational outcomes of the college & secondary households by 30% & 39.4% respectively. Downward altruism parameter (ρ) has a comparatively lesser impact, reducing human capital outcomes by 0% & 5.1% respectively. This reveals that *pen* is a more powerful predictor of educational investments than downward altruism. Downward altruism (ρ) alone fails to predict the kind of human capital outcomes typically observed in data (Azam & Bhatt (2015) and Behrman et al. (2001)).
The illiterate dad household was not making any human capital investments in the benchmark scenario ($\rho = 0.7$ & pen=1.6) so would not make any investments for lower values of intergenerational parameters either. Therefore there is no effect on this household.

Figures 7 & 8 plot the sensitivity of the portfolio choice graphs of the secondary and graduate dads for changes in dad's income, time discounting parameter, upper cap of books' investment, parental valuing of human capital parameter (pen) and downward altruism (ρ). The portfolio choice graphs are more or less stable to changes in the mentioned parameter values.

7. Conclusion

This study, through a life-cycle model, computes the optimal portfolio choice graph of a parent in the developing world in the presence of stock market entry costs & human capital investment options in the offspring. In the absence of stock market participation data in the developing world, this study fills an important gap in the literature by numerically solving the household optimization problem under borrowing constraints, income risk, and financial exclusion. It computes the benefits of financial market participation on human capital outcomes and consumption and establishes that very poor households' educational investments remain unaffected with bond or stock market participation. Only a combination of income support, as well as financial market access, can pull this category out of the intergenerational poverty trap. However, for middle-income households, human capital outcomes are more responsive to higher gains earned through market investments and increase by an average of 20%.

The study contributes to the life cycle portfolio choice literature by considering the impact of children on parental portfolio choice. It establishes the presence of two influences that act on a parent as opposed to a single worker: lower asset levels of a parent due to higher expenditures on the offspring increases stock holding appetite; flexibility in human capital investments can give a parent an additional margin of adjustment against financial risk, leading to higher stock holding as compared to a single worker.

As a rule of thumb, if human capital investments are elastic, a parent should optimally hold stock for longer periods as compared to a single worker both due to lower asset levels as well as the ability to endogenously adjust educational investments, and partially shift towards bonds a few years before his/her retirement. However, if human capital investments are inelastic, then a parent's portfolio should gradually tilt towards bond with increasing risk until the offspring separates from the parent. Thereafter, the parent should switch back to stock and gradually tilt towards bond until a few years before his/her retirement. The study also estimates that a stock market entry cost of around 1100 dollars explains the level of non-participation observed in developing countries like India, where even college educated households have 25% stock market participation. In the presence of entry costs, the study computes that the optimal entry points for households into stock markets are when the human capital investments in their offsprings are not ongoing. Higher entry costs do not alter the optimal portfolio choice graph much except delay the point of entry into the stock market. The model forwarded by this study may be extended by including fertility decisions of households.

Conflict of Interest : The author declares that there exists no conflict of interest.

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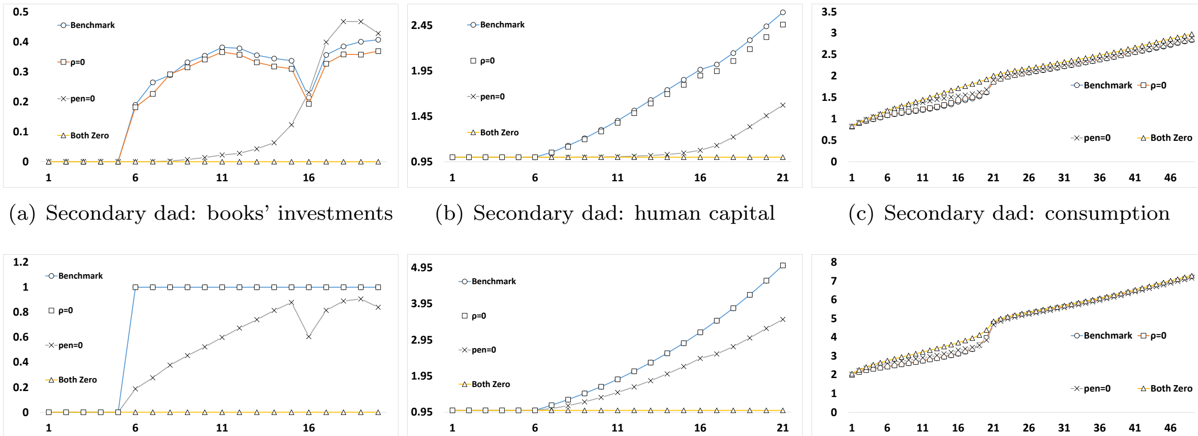
Appendices

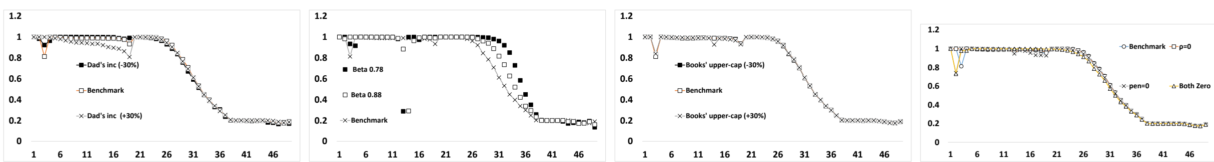
Figures



(a) Secondary dad: books' investments (b) Secondary dad: human capital (c) Secondary dad: consumption

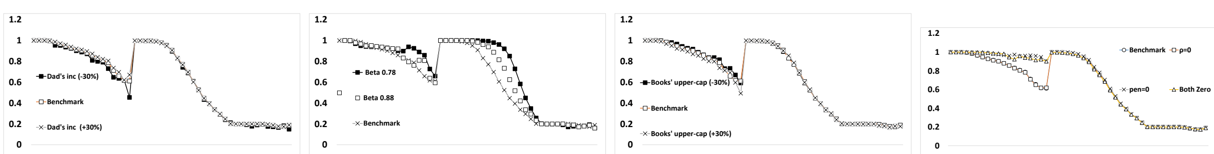
(d) College dad: books' investments (e) College dad: human capital (f) College dad: consumption

Figure 6: Sensitivity of secondary & college households' books' investments, human capital outcomes, & consumptions for different values of downward altruism coefficient ρ & parental valuing of human capital parameter *pen*



(a) Dad's income (b) Time-discounting (c) Upper-cap books (d) Downward altruism & valuing of human capital

Figure 7: Sensitivity of secondary dad's portfolio choice for a range of parameters



(a) Dad's income (b) Time-discounting (c) Upper-cap books (d) Downward altruism & valuing of human capital

Figure 8: Sensitivity of college dad's portfolio choice for a range of parameters