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WP/01/2019-20/ECO

August 2019

Measuring the Dynamics of the Achievement Gap Between Public and Private School Students During Early Life in India[*]

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August 13, 2019

Abstract

The academic achievement gap between students attending public and private schools in India is widely studied. Almost all studies so far have found evidence of private school students outperforming public school students. However, researchers have only focused on the achievement gap in levels without considering the underlying dynamics of how students move through the distribution of achievement over time. This lack of completeness is important since the extent to which policymakers and researchers should concern themselves with the public-private achievement gap should depend how mobile the students are through the test score distribution. This study aims to explore the dynamics of the public-private achievement gap in India by applying nonparametric measures of distributional mobility to panel data on math and Peabody Picture Vocabulary test scores from the Indian state of Andhra Pradesh. We find that during early childhood, public school students are at least as mobile as private school students in both upward and downward directions. However, during preadolescence, relative to private school students, public school students are significantly less upwardly mobile and more downwardly mobile. Taken together with the existence of a level gap in test scores, the mobility patterns observed in the data during the preadolescence stage suggests that one would expect to see very little convergence in the distribution of test scores as public and private school students make their way through middle and high school.

Keywords: Directional Rank Mobility, Private Schools, Public Schools, Staying Probability, Test Scores, Transition Probability

JEL Classification: I21, I24, O12, O15

[*]The authors are very grateful to seminar participants at CSSS Calcutta, ISI Delhi, Nanjing Audit Univeristy and South Asian University for insightful comments. Roychowdhury acknowledges funding under IIM Indore Seed Grant SM/08/2018-19.

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1 Introduction

The academic achievement gap between students attending public and private schools in India has been widely studied in the recent development literature (see for e.g. Kingdon, 1996; Muralidharan and Kremer, 2008; Desai et. al. 2009; French and Kingdon, 2010; Chudgar and Quin, 2012; Muralidharan and Sundaraman 2013; Singh, 2014; Singh 2015; Azam et al., 2016; ASER, 2018). Almost all studies so far have found evidence of private school students outperforming public school students in standardized tests, although there is considerable variation in the magnitude of the estimated gaps across the studies with some studies documenting large gaps in achievement while others finding evidence of only modest gaps. Summing up the findings of this literature, Singh (2014, p. 33) notes, "...private schools [in India] are associated with student achievement that are as high or higher even after accounting for all pre-existing differences in socio-economic background." Similar conclusions are also reached by Kingdon (2017. p. 28): "literature indicates that children's learning levels in private schools are no worse than, and in many studies better than, those in government schools, after controlling rigorously for the differing home backgrounds of the children in these two types of school."

Given this backdrop, this study aims to explore the dynamics of the academic achievement gap between public and private school students during early childhood and preadolescence. In particular, and using panel data on test scores in math and Peabody Picture Vocabulary Test (PPVT) for a cohort of school-enrolled children collected by the Young Lives study (YLS) in the Indian state of Andhra Pradesh, we employ two types of nonparametric measures of distributional mobility, transitional probabilities and distributional rank mobilities – that are common in the literature of income and wage mobility (Buchinsky and Hunt, 1999; Bhattacharya and Mazumder, 2011; Mazumder, 2011), to estimate the likelihood that public and private school students will transition in a directional sense (upward, downward, etc.) through the distribution of academic achievement between 5 and 12 years of age. By exploring the underlying differences in distributional mobility between private and public students, this paper contributes to the literature by providing a novel, yet complementary, measure to help better understand the progression of the public-private achievement gap during the formative years of schooling for Indian children.[1]

Understanding the dynamics of the achievement gap between public and private school students is fundamentally important for researchers and policymakers for at least two reasons. First, solely evaluating the achievement gap between public and private school students in levels and not taking into account student mobility patterns s unlikely to be meaningful and can result in both policymakers and researchers, alike, drawing incomplete conclusions regarding the seriousness of the public-private achievement gap. To see this, suppose a 'modest' achievement gap exists, yet mobilities are equally high between the students attending public and private schools. Under these conditions, it is possible that such mobility through the distribution will result in test scores being more uniformly distributed over time compared to the distribution of test scores measured at some singular point in time. In contrast, if the academic achievement gap is 'small' between public and private school students, but mobility is close to zero, then the achievement gap is permanent. What begins as an achievement gap in school has the potential to influence gaps in other important outcomes such as in skills, wages, health, and incarceration (**do we need a cite for this?**). In short, understanding the underlying dynamics of the achievement gap is needed to asses whether the 'public–private' achievement gap is transitory in nature or more of a persistent phenomena during childhood.[2]

Second, the documented public–private test score gap is unlikely to be robust to various scale transformations. In effect, the magnitude of the gap and how it evolves can vary solely

[1] We note upfront that our study is essentially descriptive in nature and may be considered as a "first pass" analysis of the dynamics of public-private achievement gap in India. Such "first pass" analysis is routinely carried out to examine the racial gaps in income mobility or gaps in academic achievement mobility in context of the US (see for e.g. Bhattacharya and Mazumder, 2011; Chetty et al. 2014; Mazumder, 2014; McDonough, 2014). While systematic investigation into the mechanisms underlying the evolution patterns documented here is important, it is outside the scope of current research, and perhaps is the next step in this research line.

[2] Similar arguments have been made by Kopczuk et al. (2010) and Glewwe (2012) in the literature on economic inequality to highlight the importance of study of income mobility.

on the basis of how underlying item responses, and thus test scores, are scaled (Bond and Lang, 2013; Jacob and Rothstein, 2016).[3] However, and as noted by McDonough (2015), distributional mobility measures are robust to monotonic scale transformations of underlying test scores so long as the rank order of students within the distribution of achievement is unchanged. Given the stable nature of distributional mobility when rank order is preserved, estimated gaps in mobility are also unchanged under such transformations. As such, scaling issues associated with achievement gaps in levels are not present when looking at gaps in distributional mobility.

Our results are compelling. We find that during early childhood, public school students are at least as mobile as private school students in both the upwards and downwards direction in math as well PPVT. During preadolescence, however, we find clear evidence of private school students being at an advantage relative to the public school students in terms of upward and downward mobility. Specifically, during preadolescence, we find that compared to private school students, public school students are significantly less upwardly mobile and more downwardly mobile in both the subjects. This indicates that the gap in academic achievement that we observe from beginning of the preadolescence period is likely to become permanent. Our results, therefore, emphasize the need for policymakers to think about smart and effective ways that could be implemented before the onset of preadolescence to promote higher upward mobility, while at the same time lower downward mobility, for public school students.[4]

The rest of the paper unfolds as follows. In the next section we discuss the prior literature. Section 3 presents the mobility measures. The data is discussed in section 4. Section 5

[3] Bond and Lang (2013), in context of the black-white achievement gap in the US, rigorously show that by selectively choosing the scale, the initial black-white gap for reading could range from one-ninth of a standard deviation all the way up to roughly half a standard deviation.

[4] It is worth emphasizing that although our findings are based on data from Andhra Pradesh, they are likely to have relevance beyond Andhra Pradesh and even beyond the Indian context. As noted by Singh (2015), the share of students enrolling in low-fee private schools has increased in several developing countries and in many of these countries (in Latin America, Asia, and Africa) these students at low-fee private schools outperform their government school counterparts. As such, findings presented here may be of importance for these other developing countries as well.

discusses the results while the last section concludes.

2 Literature Review

The proportion of private schools making up total enrollment has increased significantly over the previous two decades for both urban and rural areas in India. According to Kingdon (2017), between 2010-11 and 2015-16, student enrolment in private schools across 20 Indian states increased by 17.5 million, while that in government schools fell by 13 million; currently, private schools account for about 35% of enrolment in these 20 states.

Over the last few years, a significant body of literature has emerged that examine the learning levels of students of private schools students relative to that of students in public schools in India. One of the earliest studies on this topic examined differences in achievement between private and public school students using survey data collected in the Indian state of Uttar Pradesh (Kingdon 1996). After addressing endogenous selection into school types, the author finds that private schools provide more effective instruction in mathematics and marginally better instruction in teaching language.

Similar findings have also been reported in Tooley and Dixon (2005), Muralidharan and Kremer (2008), Desai et al. (2009), Goyal (2009) and French and Kingdon (2010). For example, Muralidharan and Kremer (2008), using survey data from the rural areas of 20 Indian states, find evidence of "private school effect" of considerable magnitude. French and Kingdon (2010), on the other hand, using ASER individual level survey data for age group 6-14 and employing village and year fixed effects, find that private school effect in basic test measures is about 0.17 standard deviation.

Among the more recent studies that examine the differential in private and public school achievement, Muralidharan and Sundararaman (2013) provide some experimental evidence of private school students achieving higher scores in Hindi and English relative to public school students. However, in Telugu (native language of Andhra Pradesh) and math there was no discernible difference in performance. When averaging the results across course subjects, the authors find that students attending private schools scored 0.23 standard deviations higher overall relative to public school students.

Using panel data from Young Lives Study in the Indian state of Andhra Pradesh, the same survey that we have drawn our data from, and employing value-added models of achievement production, Singh (2015) finds that among 8 and 10 year olds in rural Andhra Pradesh there is a positive relationship between being enrolled in a private school and English but no relationship between mathematics and private school enrollment. He also further finds that secondary school children (15 year olds) in private school in Andhra Pradesh outperform their government counterparts in mathematics by 0.2 standard deviations. However, Singh (2015) finds no impact in Telugu language.

Recently, Azam et al. (2016) also provide additional evidence of private-public schools achievement gaps from two Indian states: Orissa and Rajasthan. The authors use detailed secondary level achievement data from the two states in 2005 as part of a broader study conducted by the World Bank. Using the propensity score matching estimator, the authors find some evidence of a private school premium in Rajasthan. Specifically, the authors find that both rural and urban private school students perform higher compared to their public school peers. However, the authors find no discernible difference when looking at students in Orissa.

Chudgar and Quin (2012), using the IHDS data and corresponding achievement outcomes similar to Desai et al. (2012), find mixed results. Specifically, using regression techniques and controlling for various observables the authors find that both urban and rural private students outperform their urban and rural counterparts. However, the previous found private school premium becomes statistically insignificant after using propensity score matching to balance the data on observables between public and private students.

In sum, and at least for some subjects, the existing studies mostly find evidence of a 'private school premium' in academic achievement. None of these studies, however, compare the rates of mobility of the public school students relative to private school students over the distribution of achievement during their early life. By providing the first exploration of the dynamics of the differential in achievement from early childhood to preadolescence, our study complements the literature to provide a more complete understanding of the evolution of the achievement gap between public and private school students in India.

3 Measures of Academic Achievement Mobility

To explore the differences in achievement dynamics between public and private school students in India from early childhood to preadolescence, we borrow several metrics commonly used to measure income mobility. In what follows, we discuss each of these metrics in detail.

3.1 Probability Transition Matrices

To begin with, we construct probability transition matrices capturing the entirety of student transition dynamics over time. Specifically, let y_i^t denote the test score for student i, $i = 1, ..., N$, in time t, $t = t_0, t_1$, $t_0 \neq t_1$, and let $F_{t_0}(\cdot)$ and $F_{t_1}(\cdot)$ denote the cumulative distribution function (CDF) of test scores for students in two distinct time periods t_0 and t_1. Further, let $F_{t_0,t_1}(y^{t_0}, y^{t_1})$ denote the bivariate joint CDF, where $y^t \equiv [y_1^t \cdots y_N^t]$.

To summarize and provide intuition to the movement through the distribution of test scores captured by $F_{t_0,t_1}(y^{t_0}, y^{t_1})$, we construct a $K \times K$ transition matrix, Π_{t_0,t_1}, given by

$$\Pi_{t_0,t_1} = \begin{bmatrix} \pi_{11} & \cdots & \cdots & \pi_{1K} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \pi_{K1} & \cdots & \cdots & \pi_{KK} \end{bmatrix}. \tag{1}$$

where elements of (1) are represented by

$$\pi_{kl}^{t_0,t_1} = \frac{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0},\ \zeta_{l-1}^{t_1} \leq y^{t_1} < \zeta_l^{t_1})}{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0})} \qquad k,\ l = 1, ..., K, \tag{2}$$

and ζs are cutoff points between the K partitions such that $0 = \zeta_0^s < \zeta_1^s < ... < \zeta_{K-1}^s < \zeta_K^s = \infty$, $s = t_0,\ t_1$. Thus, $\pi_{kl}^{t_0,t_1}$ gives the fraction of children in partition k in period t_0 who are in partition l in period t_1.[5] Being completely immobile in a distributional sense implies $\pi_{kl}^{t_0,t_1}$ equals unity if $k = l$ and zero otherwise.

To derive public-private differences in these transition probabilities, we refine (2) by conditioning $X = x$, where X denotes covariates of interest. By doing such (2) simply becomes

$$\pi_{kl}^{t_0,t_1} = \int \pi_{kl}^{t_0,t_1}(x) dF(x \mid \zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0}),\ k,\ l = 1, ..., K, \tag{3}$$

where the covariate of interest is type of school (i.e., whether public or private) that a student goes to.

3.1.1 Staying Probabilities, Upward Transition Probabilities and Downward Transition Probabilities

The elements of a typical transition matrix as given by (1) can be categorized into staying probabilities (SP), upward transition probabilities (UTP) and downward transition probabilities (DTP_. For a particular k, the SP is given by $\pi_{kl}^{t_0,t_1}$, $k = l$, the set of UTP is given by $\{\pi_{kl}^{t_0,t_1}\}_{l>k}$ and the set of DTP is given by $\{\pi_{kl}^{t_0,t_1}\}_{l<k}$. For example, for $k = 3$, the SP is given by $\pi_{33}^{t_0,t_1}$, the set of UTP is given by $\{\pi_{34}^{t_0,t_1}, \pi_{35}^{t_0,t_1}, ..., \pi_{3K}^{t_0,t_1}\}$ and the set of DTP is given by $\{\pi_{31}^{t_0,t_1}, \pi_{32}^{t_0,t_1}\}$.

[5] Note, including the denominator in (2) standardizes elements of the transition matrix so that each row and column sums to one.

3.2 Directional Rank Mobilities

Useful and informative as they are, transition probabilities neither provide any information on movements *within* partitions. Accordingly, we include additional measures of mobility that track movements that students make *within* partitions, if any. Bhattacharya and Mazumder (2011) and Mazumder (2011) introduced these measures in the context of intergenerational income mobility between fathers and sons and labeled them *upward* rank mobility (URM) and *downward* rank mobility (DRM), respectively. These directional rank mobility measures capture the probability that a student's rank in the overall test score distribution when his/her age is t_1 surpasses, or falls below, his/her rank in the test score distribution at age t_0 by a constant amount.

Again let Y^{t_0} and Y^{t_1} denote a student's test score with given CDF's $F_{t_0}(\cdot)$ and $F_{t_1}(\cdot)$. URM over time, then, can be expressed by

$$\theta_{kl,\delta}^{t_0,t_1} = \frac{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0},\ F_{t_1}(y^{t_1}) - F_{t_0}(y^{t_0}) > \delta)}{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0})} \tag{4}$$

where $\delta \in [0, 1 - F_{t_0}(\zeta_k^{t_0})]$ is a predefined constant representing the threshold defining upward mobility. Similarly to (2), we can condition (4) on X by

$$\theta_{kl,\delta}^{t_0,t_1} = \int \theta_{kl,\delta}^{t_0,t_1}(x) dF(x \mid \zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0}), \tag{5}$$

where $\theta_{kl,\delta}^{t_0,t_1}(x) = \Pr[F_{t_1}(y^{t_1}) - F_{t_0}(y^{t_0}) > \delta \mid \zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0},\ X = x]$. In words, $\theta_{kl}^{t_0,t_1}(\delta)$ captures the probability of a student in the terminal period exceeding his or her initial percentile by at least δ, conditional on being located between $\zeta_{k-1}^{t_0}$ and $\zeta_k^{t_0}$in the initial period.

The same method and intuition holds for constructing DRM measures by simply reversing the logic in (4) and (5) yielding

$$\psi_{kl,\delta}^{t_0,t_1} = \frac{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0},\ F_{t_0}(y^{t_0}) - F_{t_1}(y^{t_1}) > \delta)}{\Pr(\zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0})} \tag{6}$$

and

$$\psi_{kl,\delta}^{t_0,t_1} = \int \psi_{kl,\delta}^{t_0,t_1}(x) dF(x \mid \zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0}) \tag{7}$$

where $\psi_{kl,\delta}^{t_0,t_1}(x) = \Pr[F_{t_0}(y^{t_0}) - F_{t_1}(y^{t_1}) > \delta \mid \zeta_{k-1}^{t_0} \leq y^{t_0} < \zeta_k^{t_0}, X = x]$. Again, the covariate of interest here is school-type.

To obtain the above mobility measures in practice, we first construct the empirical CDF at each time period by pooling all students (who are roughly the same age) from public and private schools for who we have valid test scores. We then find each child's ranking within the overall CDF and track how they move through the test score distribution from one time period to the next. For this paper we focus on the dynamics of public and private school students. For inference, we bootstrap the standard errors.

3.3 Directional Rank Mobilities versus Transitional Probabilities

The difference between directional rank mobilities and directional transition probabilities is directional mobilities capture movements through the distribution that are potentially neglected by staying probabilities (or directional transition probabilities). Derivation of transition probabilities requires imposing arbitrary lower and upper thresholds, ζ_{k-1} and ζ_k, that are the same for all students. Thus, in order to be counted as having 'stayed' in a particular partition, a student must fail to breach the non-unique upper and lower bounds regardless of where they initially start out in the test score distribution between ζ_{k-1} and ζ_k. Typically, a higher proportion of students counted as having 'stayed' are those who initially start out in the middle of the quantile since those students need to make larger absolute movements, up or down, compared to the students who initially started out on the peripheries of the quantile range (this is especially the case when discussing mobility patterns at the two extremes of the distribution).

Given the nature of the public-private achievement gap, it is reasonable to think that within any range of the distribution, say between the ζ_{k-1}th and ζ_kth partition, public school students will be closer to ζ_{k-1} and private school students will be closer to ζ_k, where $\zeta_{k-1} < \zeta_k$ (i.e., public school students tend to have lower test scores than private school students). Given such ordering, a measure based on arbitrarily defined cutoffs can lead to a skewed picture of mobility since some students are predisposed to not progressing/regressing simply as an artifact of their initial position in the test score distribution. Directional rank mobilities, on the other hand, are a truer sense of mobility since these measures only require a student to exceed, or fall behind, their initial rank in the distribution by some fixed amount δ. If δ is equal to zero, then any upward or downward movement in the distribution is sufficient to be counted as mobile. As such, any movement that a child makes will be captured, regardless of where they start out in the distribution of test scores.

4 Data

4.1 The Young Lives Survey

The present paper uses a sample from Young Lives Survey (YLS),[6] a longitudinal cohort study of children conducted in Andhra Pradesh between 2002 and 2014. Andhra Pradesh is the fourth-largest state in India by area and had a population of over 84 million in 2011. It is divided into three regions — Coastal Andhra, Rayalaseema and Telangana — with distinct regional patterns in environment, soil and livelihood patterns. Administratively the state is divided into districts, which are further sub-divided into sub-districts (mandals) which are the primary sampling units within our sample.[7] The sampling scheme adopted for Young Lives

[6] Young Lives project is a study on child poverty which covers four countries: Ethiopia, India, Peru and Vietnam Galab et al., (2003). For more details, please visit www.younglives.org.uk

[7] In June 2014, Andhra Pradesh was bifurcated into two states named as Andhra Pradesh and Telangana. Since then YLS continued in both the states.

was designed to identify interregional variations with the following priorities: (1) a uniform distribution of sample districts across the three regions to ensure full representation; (2) the selection of one poor and one non-poor district from each region, with district poverty classification based on development ranking; (3) when selecting poor districts and mandals, consideration was given to issues which might impact upon childhood poverty, including the presence or non-presence of the Andhra Pradesh District Poverty Initiative Programme (APDPIP).[8]

The YLS conducted a detailed and comprehensive survey of all the children divided into two cohorts: a younger cohort and an older cohort. The first survey was carried out in 2002, the second round in late 2006/early 2007, the third round was conducted from late 2009/early 2010, the fourth round took place in late 2013/early 2014, and the latest round took place in late 2016/early 2017.[9] However, the data for the latest round (2016-17) in not available in the public domain. YLS accumulated extensive information on 2,011 children who were aged 6 to 21 months (the Younger Cohort born between January 2001 and June 2002) and 1,008 children aged 7.5 to 8.5 years (the Older Cohort born between January 1994 and June 1995) for the first survey round in 2002. Information for some of the children could not be obtained after the first round due to various reasons such as death of the child, unwillingness regarding the participation in the survey, etc. In the fourth round, 1915 out of 2011 children in younger cohort and 952 out of 1008 children in older cohort were interviewed.[10]

For our study, we focus on the children who belong to the younger cohort of Young Lives.

[8] Hyderabad district is urban and metropolitan and therefore different selection criteria were applied. For more details on survey methodology see http://doc.ukdataservice.ac.uk/doc/5307/mrdoc/pdf/5307sampling_india.pdf

[9] In addition to this, school surveys for a randomly selected sub-sample taken from the younger cohort were conducted in 2010 and late 2016/early 2017.

[10] A major section of attrition occurred between the first and the second round (see Outes-Leon and Dercon, 2008 for more on attrition in the YLS).

4.1.1 Test Data

YLS collected extensive test data from children in the sample in all rounds of the survey. Test data collection was at the households of the children. As noted by Singh (2015), the tests differed in their focus on which dimension of cognitive achievement they attempted to capture and how closely they related to the formal school curriculum in Andhra Pradesh; often, different tests were administered to children across rounds in order to ensure that they were appropriate for the age and the stage of education that the children were in. For children belonging to the younger cohort, YLS conducted math[11] and PPVT[12] in all rounds of the survey except the first round, since the children were too young (6 to 18 months old) to respond to any academic test. Additionally, a writing (English) test was introduced from the third round whereas a language test was conducted in the fourth round.

Note, the tests used in Young Lives are much more comprehensive in the domains of learning they capture and offer more variation than tests in most previous studies in the literature,which is a considerable strength of the data. In particular, as noted in Singh (2015), the ASER data collected by Pratham and the India Human Development Survey 2005 (used in Chudgar and Quin, 2012; Desai et. al., 2008; French and Kingdon, 2010) have only very basic test measures, on a more limited set of domains, which are not best-suited to capturing variation across the ability spectrum.

4.2 Analytic Sample

Using the YLS test data, we estimate transition probabilities and directional rank mobilities for public and private school student for the two periods, from 2006/07 to 2009/10 (when the children were between 5 and 8 years of age) and from 2009/10 to 2013/14 (when the

[11] In the second round, Cognitive Development Assessment (CDA) is conducted where quantity based questions are asked from the children. A sample question of CDA is "Look at the plates of cupcakes. Point to the plate that has a few cupcakes... Point to the plate that has a few cupcakes". We consider this test also as a form of Mathematics test as knowledge of numbers is required to answer to the questions in CDA.

[12] The PPVT was initiated in 1959 to analyze the verbal intelligence of a child. It also helps in evaluating the scholastic aptitude for the children in school going age.

children were between 8 and 11 years of age). In other words, we examine the evolution of student achievement from early childhood to the end of preadolescence. We consider test scores of children in math and PPVT since these are available for all the three rounds. For both math and PPVT, we convert the raw test scores to item response theory (IRT) test scores.[13][14]

4.2.1 Sample Construction

In 2006/07, 1950 children were surveyed. Out of them 863 report to have been attending school. We however have information on the type of school the students go to for 789 children. Out of these 789 children, 595 were in public school and 194 were in private school. In 2009/10, 1930 children were surveyed. Of these 1930 students, 1051 were in public school and 863 were in private school, 8 children were not going to school regularly or not going to school at all, while the school type information is missing for the remaining 8 children.[15] In 2013/14, 1915 children were surveyed. Out of 1915 children, information on the type of school they attend is available for 1858 children. Out of these 1858 children, 1102 were in public school and 756 were in private school.[16]

For our analysis, we focus only on the sample of students who did not drop out from school or who did not switch from a public to a private school or from a private to a public

[13] IRT model posits a mathematical relationship between latent ability and observed responses to test scores. IRT model is routinely employed in educational assessment (for e.g. by international testing programs like PISA, TIMSS, GRE and SAT).

[14] Our analysis doesn't use any sample weights as there are no survey weights available in public use YLS data.

[15] There are 1132 students who were not in school in second round but were in school by the next round. Out of these 1132 students, 573 were in private and 559 in public school in third round. (How are we getting the figure of 1132??)

[16] Note, in our analysis, while public schools refer to pure public schools, private school students refer to children going to pure private schools, community schools (run by NGOs, charitable organizations, religious organizations, etc.) and public aided private schools. However, in all the rounds, among the students classified as private school students, the majority are indeed students of pure private schools. Specifically, in the 2006/07 wave, 87% of children classified as private school students, attend pure private schools, in 2009/10 wave, 98% of students classified as private school students attend pure private schools, and in 2013/14 wave, 82% of students classified as private school students attend pure private schools.

school between two successive rounds of the survey.[17] Out the 595 school students who were public schools in 2006/07, 124 students moved to private schools and 4 students dropped out between 2006/07 and 2009/10. Thus, the number of public school students who remained in the public school between 2006/07 and 2009/10 is 467. Of the 194 students who were in private schools in 2006/07, 25 students moved to public schools and 3 students dropped out between 2006/07 and 2009/10. Thus, the number of public school students who remained in the private school between 2006/07 and 2009/10 is 166. Thus, our anlytic sample for the analysis of the mobility dynamics during early childhood includes 467 public school students and 166 private school student.

Out of the 1051 children who were in public schools in 2009/10, 91 students moved to private schools and 54 students dropped out of school between 2009/10 and 2013/14. Thus, the number of public school students who remained in the public school between 2009/10 and 2013/14 is 906. Of the 863 students who were in private schools in 2009/10, 186 students moved to public schools and 14 students dropped out between 2009/10 and 2013/14. Thus, the number of public school students who remained in the private school between 2006/07 and 2009/10 is **663**. Our analytic sample for the analysis of mobility dynamics during children's preadolescence consists of 906 public school students and 663 private school students.[18]

The summary statistics of our analytical sample is presented in Table 1. The kernal density distribution of test scores in math and PPVT for the three different rounds of survey are presented in Figure A1 in the Appendix

[17] We take this conservative route of dropping the switchers and dropouts. If, instead, we had included them and considered our public (private) school sample to consist as all those who were attending public (private) school just in the initial period immaterial of the fact that whether they continued to be in the public (private) school till the next survey, then while calculating mobility estimates for public (private) school would end up considering all those who potentially might have switched school or dropped out of school right after the initial survey. By not being able to get rid of these students, our mobility estimates for the two school types might be measured with error.

[18] [Later on, we will use the sample of students who did not drop out from school or who did not switch from a public to a private school or from a private to a public school for all three rounds of the survey. Non-movers for all the three rounds: 414 public school students and 124 private school students.]

5 Results

We now turn to the analysis of the achievement gap between public and private school students using various mobility metrics introduced in Section 3. We begin with the analysis based on the transition matrices. We then discuss the results obtained using directional rank mobility measures.

5.1 Transition Matrices

5.1.1 SP

The estimates of SP for public and private school students in math and PPVT are plotted in Figure 1. In both the figures, panel A shows the staying probabilities probabilities estimated using the 2006/07 and 2009/10 waves of the data, whereas panel B shows the transition probabilities estimated using the 2009/10 and 2013/14 waves of the data. Thus, panel A depicts staying probabilities during early childhood, while panel B shows the estimates of staying probabilities during preadolescence. The x-axis varies the sample used based on the quartile range of IRT test score, while the y-axis shows the staying probability. The green lines show the estimates for the public school students, while the maroon lines show the estimates for private school students. The black line plots the difference in the probabilities by school type, along with standard error bands.

We begin by discussing the results for math. It turns out that, compared to the private school students, public school students have 14 percentage points (p.p.) higher chances of staying at the lowest quartile of test score distribution during early childhood and 12 p.p. higher probability of staying at the lowest quartile of test score distribution during preadolescence. However, the difference is statistically significant only during preadolescnce. As we move to the right and gradually increase the quartile range of test scores, the difference in staying probabilities between public and private school students falls and becomes negative in the top two quartiles. At the highest quartile of test score distribution, we find that, compared to public school students, private school students are 3 p.p. more likely to stay during early childhood and 34 p.p. more likely to stay during preadolescence. Again, the difference is statistically significant only during preadolescence.

For PPVT, we observe a more or less similar pattern. Compared to the private school students, public school students have 8 p.p. higher chances of staying at the lowest quartile of test score distribution during early childhood and 25 p.p. higher probability of staying at the lowest quartile of test score distribution during preadolescence. The difference is statistically significant during preadolescence but not during early childhood. As we move to the right, the difference in staying probabilities between public and private school students falls and becomes negative in the third and fourth quartiles. We find that private school students, compared to the public school students, are 27 p.p. more likely to stay at the top of test score distribution during early childhood and 18 p.p. more likely to stay at the top of test score distribution during preadolescence. However, unlike in math, the estimate of both these differences highly statistically significant.

5.1.2 UTP

Figure 2 plots an analogous set of estimates of upward transition probabilities for public and private school students in math and PPVT . We plot the estimates of the upward transition probabilities (and the difference) for only those public and private school students whose test score were at or below the first quartile in the initial period. Estimates of the upward transition probabilities of students whose test score were in the second or third quartiles in the initial period can be found in Tables A1 and A2 in the Appendix.

In math, we observe that UTP for public school students is much smaller than the UTP for private school students across all the quartiles during early childhood. For example, the likelihood of a public school student moving upwards from the first to the third quartile is 15 p.p.; the corresponding figure for a private school student is 19 p.p.. However, note that the none of the differences in UTP are statistically significant. During preadolescence, also, two out of the three differences in UTP are statistically insignificant. The only statistically significant difference in UTP between public and private school students is observed in case of movement from the first to the second quartile with UTP of private school students being higher than that of public school students by 9 p.p.

Next we turn to the comparison of UTP between public and private school students in PPVT. During early childhood, we find no statistical difference in the rates of upward mobility between the students from public and private schools residing in the bottom quartile of the test score distribution when considering movement to the second or third quartiles over time. We, however, observe that the difference in UTP is statistically significant when considering movement from the lowest to the highest quartile with private school students having 22 p.p. higher UTP compared to public school students. During preadolescence, we find the public school students have significantly lower UTP compared to private school students when considering movement between first and third or fourth quartiles. For example, in the beginning of preadolescence while the first quartile public school students had 11 p.p. and 4 p.p. probability of reaching the third or fourth quartiles by the end of preadolescence, the corresponding figures for the private school students were 29 p.p. and p.p. However, when considering movement from the first to the second quartile, the public school students seem to have a slight advantage over private school students, although the difference in UTP in this case is not statistically significant.

5.1.3 DTP

Figure 3 plots the downward transition probabilities. We plot the estimates of the downward transition probabilities (and the difference) for only those public and private school students whose test score were in fourth quartile in the initial period. Estimates of the DTP of students whose test score were in the second or third quartiles in the initial period can be found in Tables A1 and A2 in the Appendix.

Looking at the DTP for math, we notice that for both public and private school students, the DTP declines as we move towards the right during early childhood with the DTP for public school students being always at least as high as the DTP for private school students. However, the difference in the DTPs between public and private school students are never statistically significant. During preadolescence also, for both public and private school students, the DTPs declines as we move towards the right during early childhood and the DTP for public school students are always higher than the DTP for private school students. However, now the difference in the DTPs between public and private school students are always highly statistically significant. For example, the public school students, compared to the private school students, are 14 p.p., 15 p.p. and 5 p.p. more likely to transition out from the top quartile to the third quartile, second and first quartiles respectively.

For PPVT, our results are somewhat similar to the results for math during early childhood. Specifically, we do not find any evidence of any statistically significant difference in DTP between public and private school students who reside in the top quartile of the test score distribution. For preadolescence, however, our results are different from what we have obtained for math. In PPVT, unlike math, public school students do not seem to always have a greater chance of sliding down when at the top. For example, compared to private school students, the public school students although have 9 percentage points greater probability of moving from the fourth to the second quartile, but have almost the equal probability of moving down to the very bottom of the test score distribution.

5.2 Directional Rank Mobilities

5.2.1 URM

Figures 4 plots the estimates of URM when $\delta = 0$ and figure 5 plots the estimates of URM when $\delta = 0.10$. These estimates, along with that of DRM, are also available in a tabular form in the Appendix (Tables A3 and A4). We begin by discussing the URM estimates for math. We find that during early childhood, when we set $\delta = 0$, the URM estimates of public school students and private school students are very similar with none of the differences exhibiting statistical significance. When we set, $\delta = 0.10$, as expected, the URM estimates of public school students as well as private school students fall compared to the case when $\delta = 0$, and the differences between the two are even smaller (and not statistically significant as before). During preadolescence, however, our results are dramatically different. When $\delta = 0$, we observe that the URM estimates for private school students always significantly exceeds the URM estimates for public school students. The differences are large, ranging between 10 to 29 p.p., as well as statistically significant. When we set $\delta = 0.10$, the URM estimates for both the public school students and the public school students as well as the differences between them fall slightly. However, three out of the four differences continue to remain statistically significant.

We next turn our attention to the estimates of URM for PPVT. The results are very similar to what we have obtained for math. During early childhood, as in case of math, we do not find evidence of any difference in the URM estimates between private and public schools in any of the quartiles of the test score distributions. This holds true for $\delta = 0$ as well as $\delta = 0.10$. However, during preadolescence, we find clear evidence of private school students being significantly more upwardly mobile than public school students across all the quartiles of the test score distribution when $\delta = 0$ and in three out of four quartiles when $\delta = 0.10$.

5.2.2 DRM

Next we turn to the estimates of DRM. Figure 6 plots the estimates of DRM when $\delta = 0$ and figure 7 plots the estimates of DRM when $\delta = 0.10$. For math, our findings are as follows. During early childhood, the DRM estimates of public school students and private school students are not significantly differently in any of the quartiles. When we set, $\delta = 0.10$, the DRM estimates of public school students as well as private school students fall compared to the case when $\delta = 0$, and the differences between the two reduces across all the quartiles compared to the case when $\delta = 0$. However, during preadolescence, when $\delta = 0$, the DRM estimates for public school students always seem to significantly exceed the DRM estimates for private school students. The differences are remarkably large as well as statistically significant. When we set $\delta = 0.10$, the DRM estimates for both the public school students and the public school students fall slightly compared to the case when $\delta = 0$; the differences in DRM between public and private school continue to remain large and statistically significant across all the quartiles.

For PPVT, as in math, we do not find evidence of any difference in the DRM estimates between private and public schools in any of the quartiles of the test score distributions during early childhood. This holds true for $\delta = 0$ as well as $\delta = 0.10$. During preadolescence, public school students, compared to private school students, seem to have significantly higher chances of moving downwards across all the quartiles of the test score distribution when $\delta = 0$. When $\delta = 0.10$, the magnitude of the differences in the DRM remains almost the same compared to the case when $\delta = 0$, although the differences in the bottom and top quartile of the test score distribution fail to exhibit statistical significance.

5.3 Implications of the gap in achievement between public and private school students

As discussed above, the mobility patterns of public and private school students appear to be similar during early childhood but divergent in nature during preadolescence. Indeed, during preadolescence, we observe a pattern of public school students falling down through the distribution of achievement and at the same time struggling to climb up through the distribution compared to private school students. The natural question is, therefore, what are the implications of such mobility patterns?

To examine this question, using the matrix of transition dynamics for both public and private school students, we derive the Markov chain steady-state distributions given the underlying mobility patterns using standard matrix algebra. We do this separately for the early childhood phase as well as preadolescence phase. This would allow us to examine the public-private gap in academic achievement assuming that the estimated transition probabilities are a permanent feature of the educational system in Andhra Pradesh. The Markov-chain steady-state distributions, by subject and school type, are displayed in Figure 8.

The results show that in steady state, in math, if the estimated transition probabilities for early childhood persisted in the long run, there would not be much difference in the proportion of public and private school students settling across the different quartiles. For example, while 27% of public school students would settle at the lowest quartile of the test score distribution, around 20% the private school students would settle in the same quartile in the long run. On the other hand, around 23% of public school students would settle in the highest quartile of the test score distribution compared to 28% of the private school students. Things, however, are dramatically different if we assume that the transition probabilities estimated for preadolescence persist in the long run. In this case, we find that around 35% of the public school students settle in bottom quartile of the test score distribution as compared to only 11% of private school students. On the other hand, 47% of the private school students settle in the top quartile as compared to 13% of public school students.

For PPVT also, the difference in the proportion of public and private school students settling across the different quartiles of the test score distribution are relatively more uniform during early childhood as compared to preadolescence. For example, during early childhood, around 28% of public school students and 18% of private school students settle in the lowest quartile. In the highest quartile, the corresponding figures are21% and 38% respectively. During preadolescence, around 32% of the public school students settle in the bottom quartile, compared to 16% of the private school students. In the top quartile, the proportion of public and private school students settling are 16% and 38% respectively.

These results suggest that the mobility patterns prevailing during preadolescence are likely to lead to large and significant gap in achievement between public and private school students in the long run. The mobility patterns prevailing during early childhood, on the other hand, might not give rise to significant gap in academic achievement between public and private school students.

5.4 Conditional Mobility Gap

On the face of things, our results indicate private (public) school students are more upwardly (downwardly) mobile than public (private) school students at least during preadolescence. However, this might not be a fair comparison because of the self-selection associated with children who attend private schools. As noted by Wadhwa (2018, p. 18), "it is well known that children who go to private schools come from relatively affluent backgrounds and tend to have more educated parents. This affords them certain advantages that aid learning. These advantages are not available to children who are from less advantaged families and are more likely to attend government schools." In the preceding analysis, we have not controlled for any factor which could be potentially determining children's selection into schools. If we control for these factors that affect learning, the mobility gap in math or PPVT levels during preadolescence between children attending different types of schools might disappear. In order assess whether this is the case (in other words, whether self selection into schools are driving our results), we implement a linear probability model (LPM) and estimate the mobility gap between public and private schools conditional on various covariates that could be potentially influencing children's decision to attend a public or a private school. Note, conditioning mobility measures on multiple covariates concomitantly is difficult to do nonparametrically given the small sample sizes that arise as more and more covariates are introduced.

Specifically, to examine the conditional URM, we estimate

$$y_{it} = \alpha + \gamma S_i + \mathbf{x}_{it}\beta + \varepsilon_{it} \tag{8}$$

where y_{it} is equal to 1 if $F_t(Y_t) - F_{t-1}(Y_{t-1}) > \delta$, 0 otherwise; S_i is a school dummy equal to 1 if school type is public, 0 otherwise; $\mathbf{x}_{it}$ is a vector of covariates, ε_{it} is an i.i.d. error term and $t = 1, 2$. As such, the estimate for γ is equal to the public-private gap in URM controlling for $\mathbf{x}_{it}$. To assess the gap in the DRM conditional on various covariates, we simply construct y_{it} equal to 1 if $F_{t-1}(Y_{t-1}) - F_t(Y_t) > \delta$ and 0 otherwise. We estimate (x) separately for both math and PPVT.

The specific covariates that used in this exercise include the child's gender, family size, caste affiliation, household wealth index and a dummy indicating whether the child lives in a urban or rural area. All estimates for the public-private gap in directional mobility measures, as well as the point estimates for the various observables, can be found in Tables 2 and 3.

We find that during preadolescence, conditional on the covariates, public school students are significantly less mobile in the upward direction and significantly more mobile in the downward direction than the private school students in math as well as PPVT. Specifically, when $\delta = 0$, compared to private school students, public school students have 15 percentage points less probability of moving upwards and have 15 p.p. more probability of moving downwards in the distribution of test score in math. When $\delta = 0.10$, these gaps reduce slightly: public school students now are 13 p.p. less likely to move upwards and 11 p.p. more likely to move downwards than private school students. For PPVT, the results are somewhat similar. When $\delta = 0$, compared to private school students, public school students have 7 p.p. less probability of moving upwards and have 7 p.p. more probability of moving downwards. When $\delta = 0.10$, compared to private school students, public school students have 9 p.p. less probability of moving upwards although the probability of moving being upwardly mobile does not significantly differ between public and private school students significantly.

For early childhood, our findings however indicate no superiority of private school students over public school students in terms upward or downward mobility. In fact, in both math and PPVT, private school students, compared to public school students, appear to be less upwardly mobile and more downwardly mobile. For example, when $\delta = 0$, in math, private school students are 17 p.p. less likely to move in the upwards direction and 19 p.p. more likely to move in the downwards direction compared to public school students. In PPVT, private school students, compared to public school students, are 10 p.p. less likely to move in the upwards direction and 10 p.p. more likely to move in the downwards direction. The results are similar when $\delta = 0.10$. These results are in sharp contrast to the results that we have obtained for the preadolescence phase.

Coupled with the previous findings, these results suggest that during preadolescence, compared to public school students, the private school students seem to be significantly more upwardly mobile and significantly less downwardly mobile. During early childhood, however, we do not find any evidence of private school students having higher chances of moving upwards and lower chances of moving downwards compared to the public school students. In fact, our analysis of conditional mobility gaps indicate that during early childhood, if anything, the public school students are more upwardly and less downwardly mobile compared to the private school students.

6 Conclusion

The academic achievement gap between students attending public and private schools in India has been widely studied. Almost all studies so far have found evidence of private school students outperforming public school students in standardized tests. However, researchers have only focused on the achievement gap in levels without considering the underlying dynamics of how students move through the distribution of achievement over time. However, not taking into consideration the underlying mobility patterns of students can lead to incorrect conclusions regarding the severity of the gap in academic achievement. If mobilities are equally high between public and private school students, then it is likely that the dispersion of test scores between the two groups will get smoothed out over the entire distribution of test takers as students advance their way through school. However, if students are extremely immobile, then even the slightest disparity in test scores can be thought to be as worrisome since the gap in levels, to at least some degree, is permanent.

In this study we explore the dynamics of the public-private achievement gap in India by applying nonparametric measures of distributional mobility to panel data on math and Peabody Picture Vocabulary test scores from the Indian state of Andhra Pradesh. We find that during early childhood, public school students are as mobile as (if not more than) private school students in both the upwards and downwards. direction. However, during preadolescence, relative to private school students, public school students are significantly less upward mobile and more downwardly mobile. Given the steady-state distributions associated with derived transition matrices during preadolescence, we show that if the observed differences in mobility were allowed to persist indefinitely, then the gap in academic achievement between public and private school students that is observed from the beginning of preadolescence is likely to remain a permanent problem in school education system in India.

Our findings in no way suggest that the convergence in academic achievement between public and private school students is impossible. Instead, our results emphasize the need for policymakers to think about smart and effective interventions that could be implemented before the onset of preadolescence to promote higher upward mobility, while at the same time lower downward mobility, for public school students. Finally, we would also like to emphasize that as with simply focusing on the level gap in test scores between public and private school students, solely focusing on the mobility measures outlined in this paper would be equally misleading. Rather taking into account both the level differences in performance between public and private school students, as well as differences in mobility measures, better equips policymakers with the insights needed to design effective education-based interventions relative to any one measure alone.

References

[1] Azam, M., Kingdon, G., and Wu, K. B. (2016). "Impact of private secondary schooling on cognitive skills: evidence from India," *Education Economics*, 24, 465-480.

[2] Bhattacharya, D., and Mazumder, B. (2011). "A nonparametric analysis of black-white differences in intergenerational income mobility in the United States," *Quantitative Economics*, 2, 335–379.

[3] Buchinsky, M., and Hunt, J. (1999). "Wage mobility in the United States," *Review of Economics and Statistics*, 81, 351–368.

[4] Chetty, R., Hendren, N., Kline, P., Saez, E. and Turner, N. (2014). "Where is the land of Opportunity? The Geography of Intergenerational Mobility in the United States," *Quarterly Journal of Economics*, 129, 1553–1623.

[5] Chudgar, A., and Quin, E. (2012). "Relationship Between Private Schooling and Achievement: Results from Rural and Urban India." *Economics of Education Review*, 31, 376-390.

[6] Desai, S., Dubey, A., Vanneman, R., and Banerji, R. (2009). "Private schooling in india: A new educational landscape," In *India Policy Forum,* 5, 1-58.

[7] French, R. and Kingdon, G. (2010). "The relative effectiveness of private and government schools in Rural India: Evidence from ASER data," Department of Quantitative Social Science Working paper, 10-03.

[8] Galab, S., Reddy, M. G., Antony, P., McCoy, A., Ravi, C., Raju, D. S., Mayuri, K., & Reddy, P. P. (2003). "Young lives preliminary country report: Andhra Pradesh, India," Young Lives Project, London, September.

[9] Glewwe, P. (2012). "How Much of Observed Economic Mobility is Measurement Error? IV Methods to Reduce Measurement Error Bias, with an Application to Vietnam," *World Bank Economic Review*, 26, 236-264.

[10] Goyal, S. (2009). "Inside the house of learning: the relative performance of public and private schools in Orissa," *Education Economics*, 17(3), 315-327.

[11] Kingdon, G. G. (1996). "The Quality and Efficiency of Public and Private education: A case study of Urban India," *Oxford Bulletin of Economics and Statistics*, 58 (1), 57–82.

[12] Kingdon, G.G. (2017). "The Private Schooling Phenomenon in India: A Review," IZA DP No. 10612.

[13] Kopczuk, W., Saez, E., and Song, J. (2010), "Earnings Inequality and Mobility in the United States: Evidence from Social Security Data Since 1937," *Quarterly Journal of Economics*, 125, 91-128.

[14] Mazumder, B. (2011). "Black-white differences in inter generational economic mobility in the U.S.," FRB of Chicago Working Paper, No.2011-10.

[15] McDonough, I.K. (2015). "Dynamics of the black–white gap in academic achievement," *Economics of Education Review*, 47, 17-33.

[16] Muralidharan, K., and Kremer, M. (2008). "Public and Private Schools in Rural India." in Chakrabarti, R. and Petersen, P. (eds.), *School choice international: Exploring public–private partnerships*, Boston, MA: MIT Press.

[17] Muralidharan, K. and Sundararaman, V. (2013). "The Aggregate Effect of School Choice: Evidence from a Two-Stage Experiment in India." *Quarterly Journal of Economics*, 130, 1011-66.

[18] Outes-Leon, I., and Dercon, S. (2008). "Survey attrition and attrition bias in Young Lives," Young Lives Technical Note 5. Young Lives, University of Oxford.

[19] Pratham (2005). *Annual Status of Education Report*, New Delhi: Pratham Documentation Center.

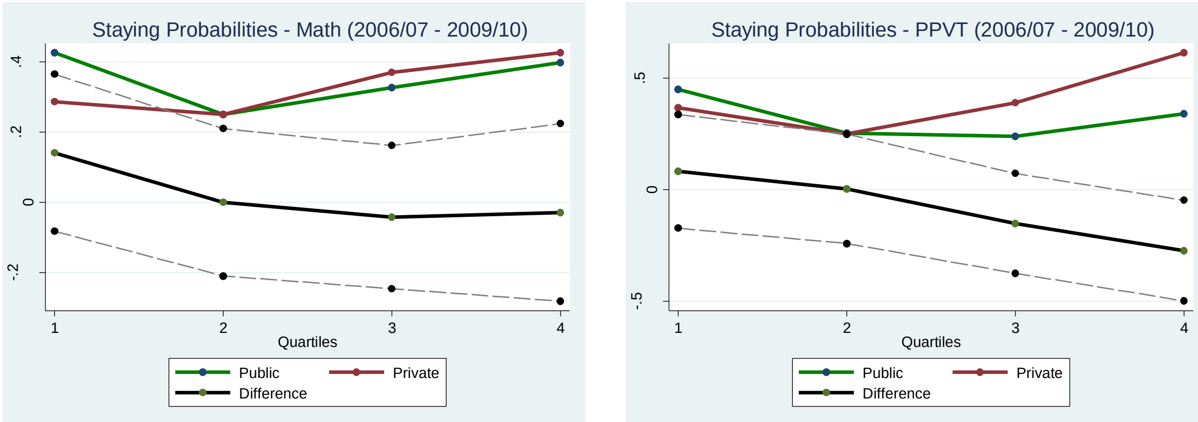
[20] Singh, A. (2014). "Test score gaps between private and government sector students at school entry age in India," *Oxford Review of Education*, 40, 30-49.

[21] Singh, A. (2015). "Private school effects in urban and rural India: Panel estimates at primary and secondary school ages", *Journal of Development Economics*, 113 C, 16-32.

[22] Tooley, J., and Dixon, P. (2005). "Private School is Good for the Poor: A Study of Private Schools Serving the Poor in Low-Income Countries." CATO Institute, Washington, DC.

[23] Wadhwa, W. (2018). "Equity in learning?," *Annual Status of Education Report (Rural) 2018*, 17–19, Pratham, New Delhi, India.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

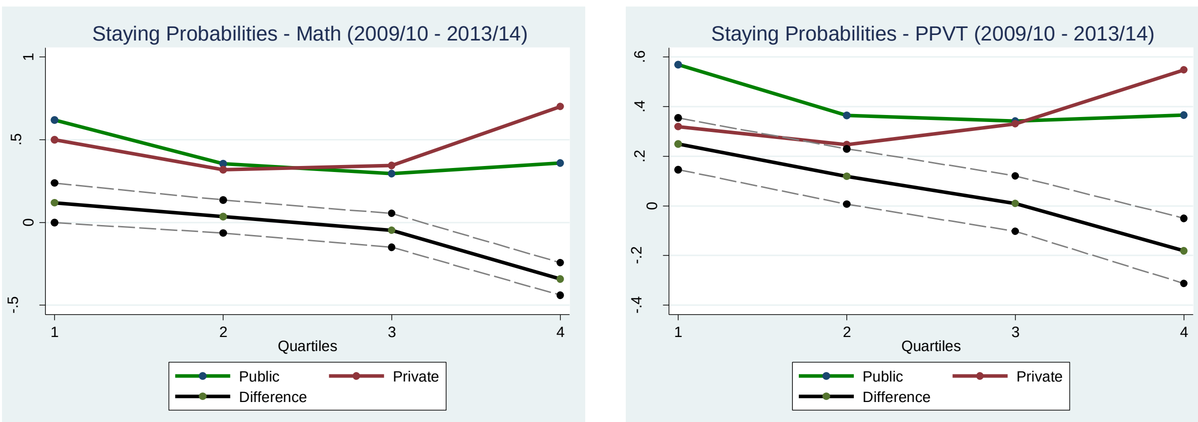
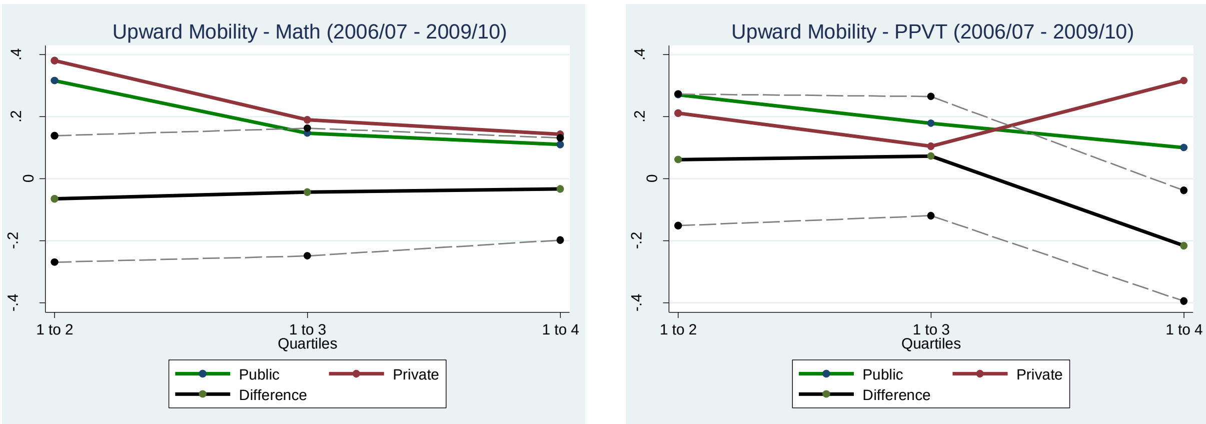


Figure 1. Staying Probabilities

Notes: Y-axis shows estimated staying probabilities. The dotted lines represent 95% confidence interval for the estimated difference between public and private staying probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

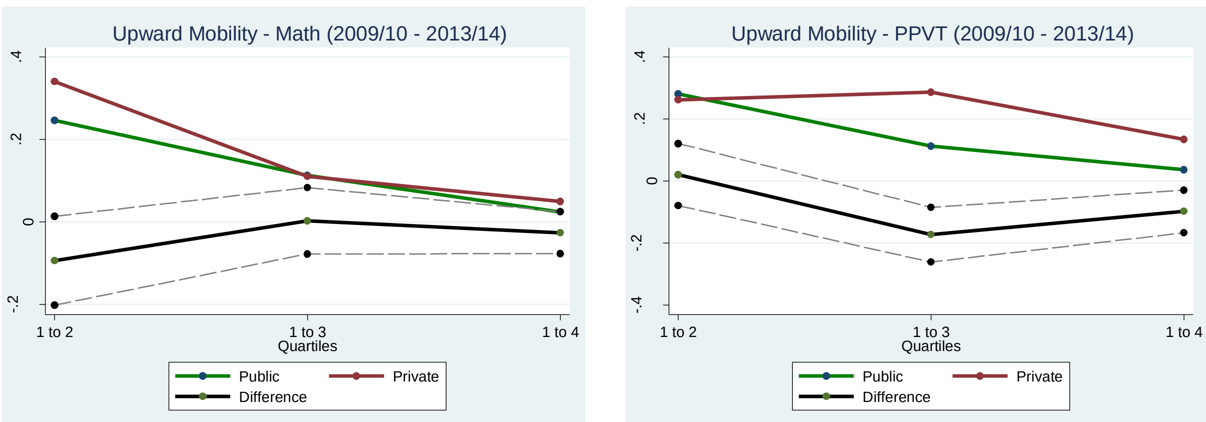
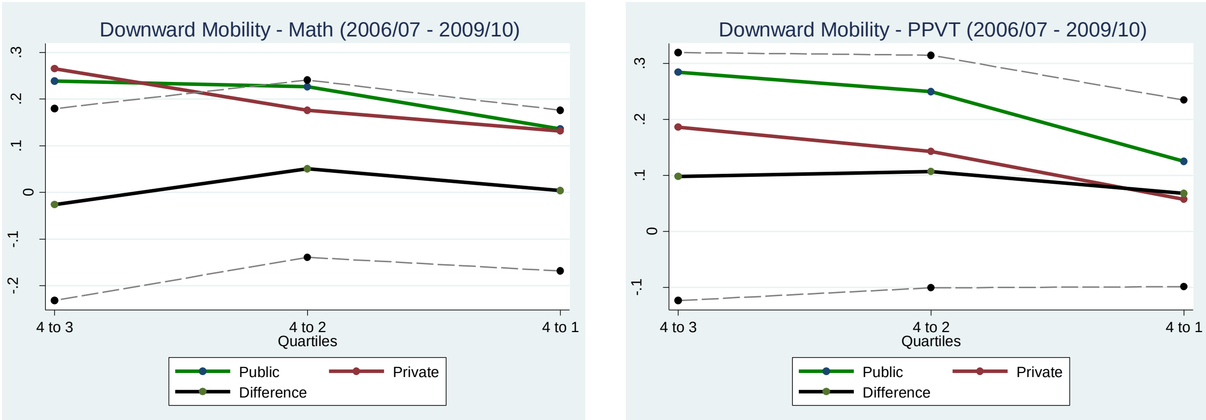


Figure 2. Upward Transition Probabilities

Notes: Y-axis shows estimated transition probabilities and X-axis shows movement from quartile 1 in the initial period to some higher quartile in the final period. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

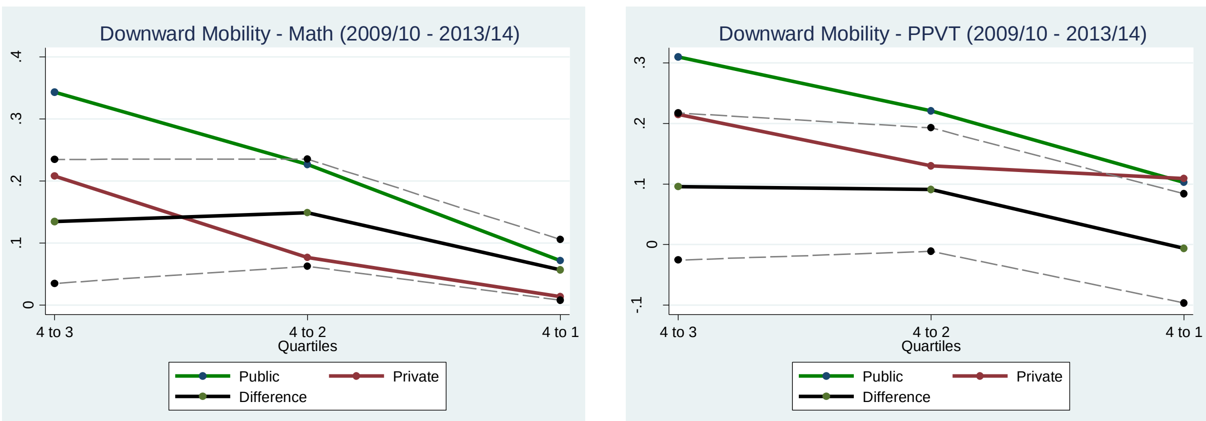
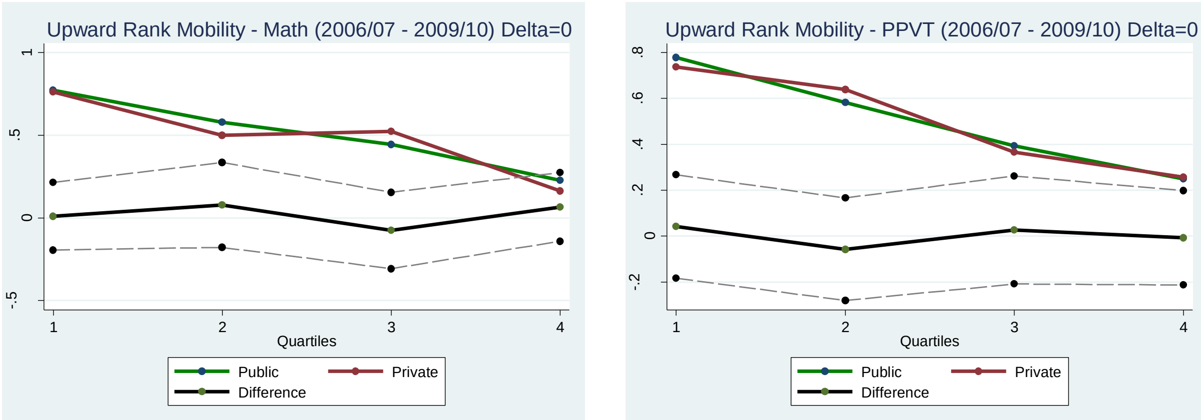


Figure 3. Downward Transition Probabilities

Notes: Y-axis shows estimated transition probabilities and X-axis shows movement from quartile 4 in the initial period to some lower quartile in the final period. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

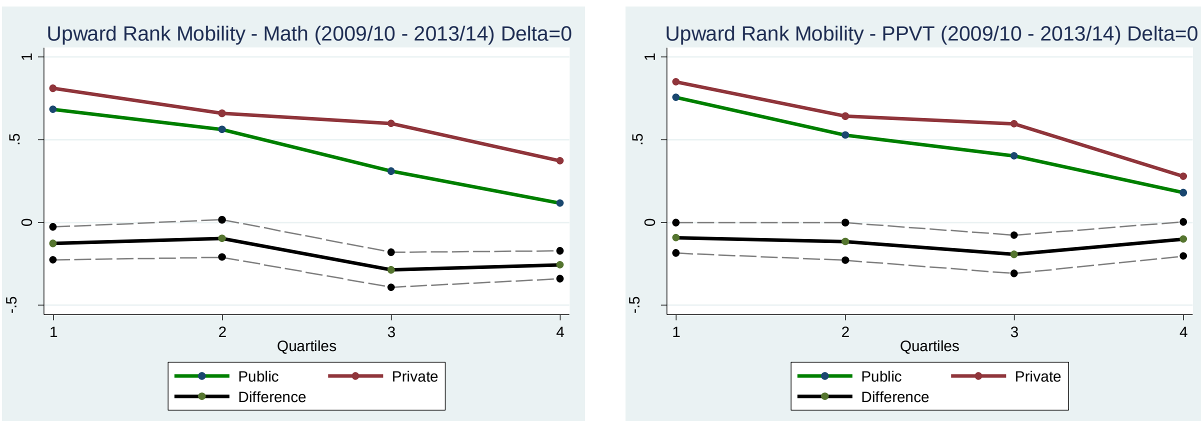
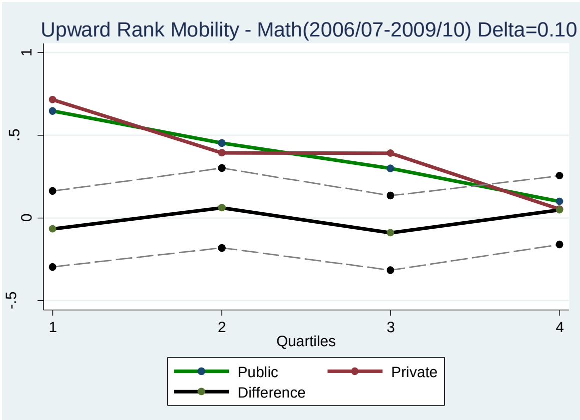
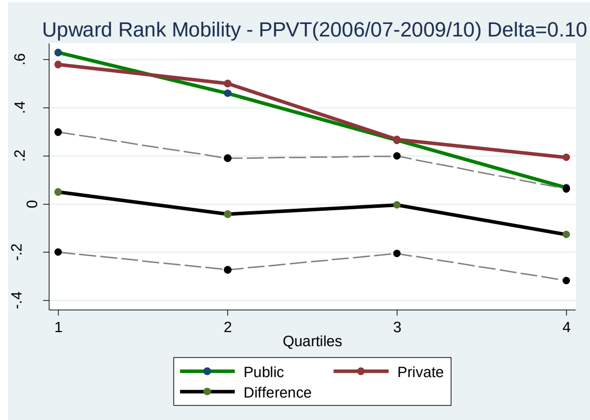


Figure 4. Upward Rank Mobility (δ = 0)

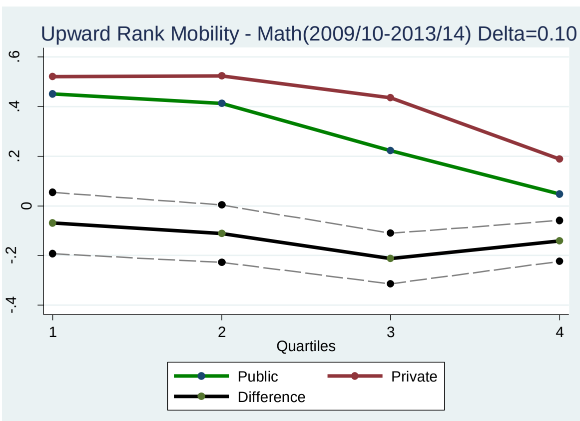
Notes: Y-axis shows estimated transition probabilities. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



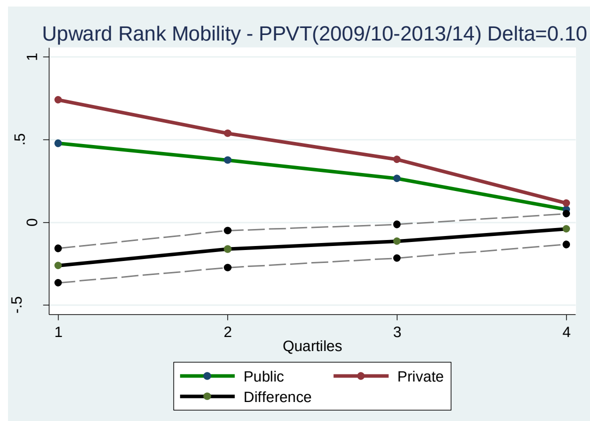
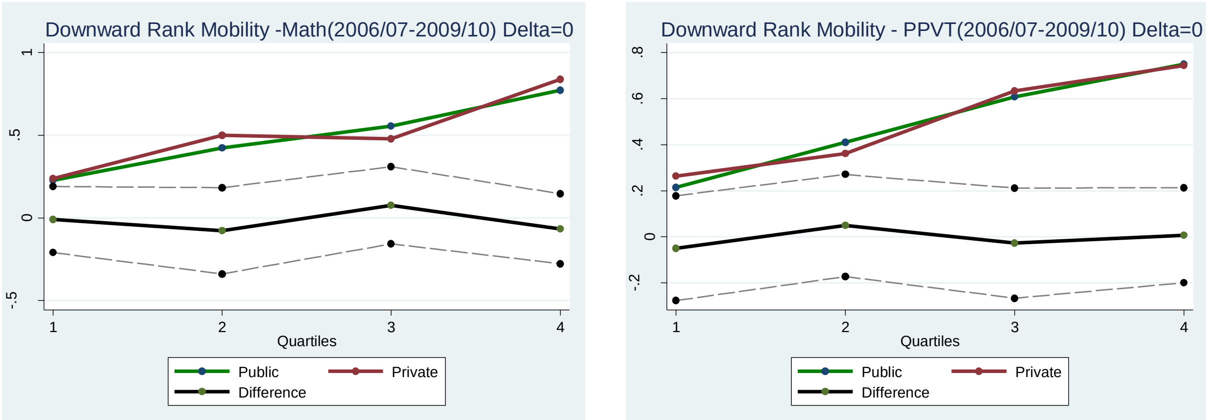


Figure 5. Upward Rank Mobility (δ = 0.10)

Notes: Y-axis shows estimated transition probabilities. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

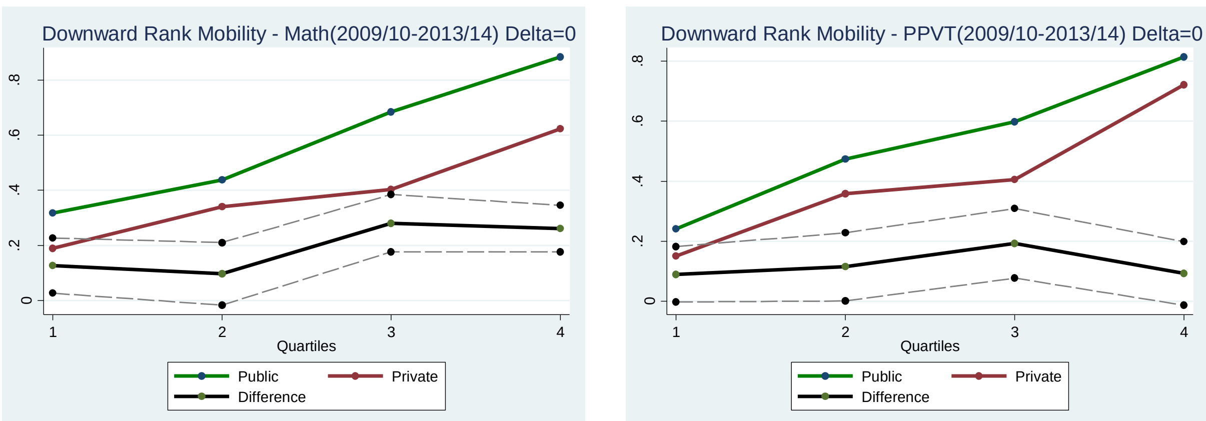
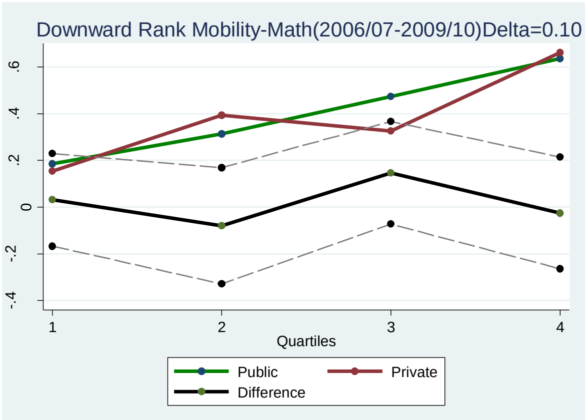
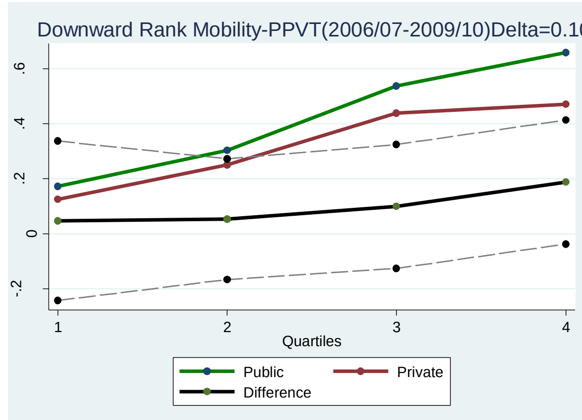


Figure 6. Downward Rank Mobility (δ = 0)

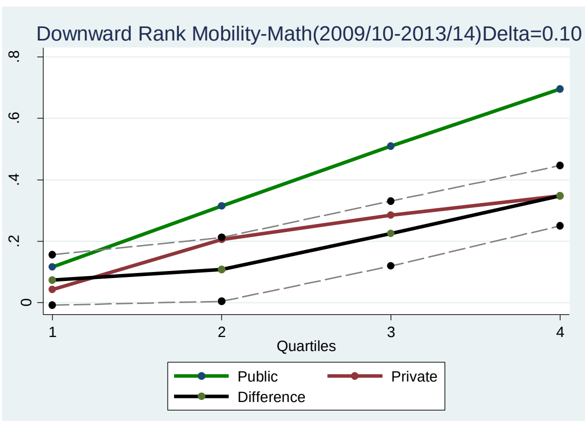
Notes: Y-axis shows estimated transition probabilities. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



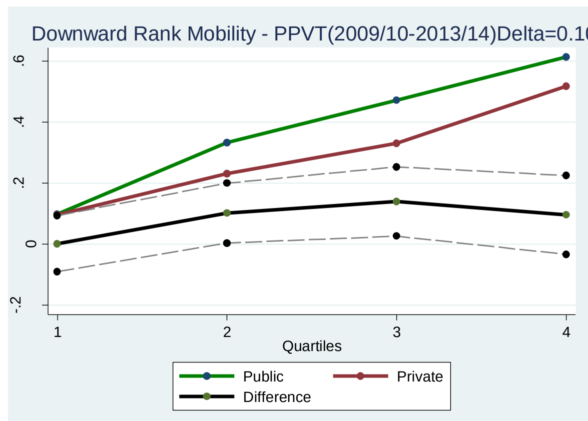
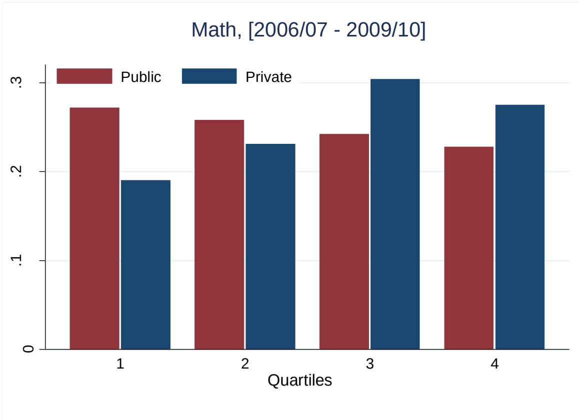
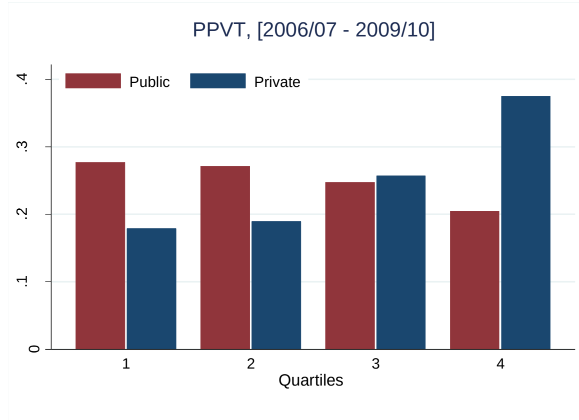


Figure 7. Downward Rank Mobility (δ = 0.10)

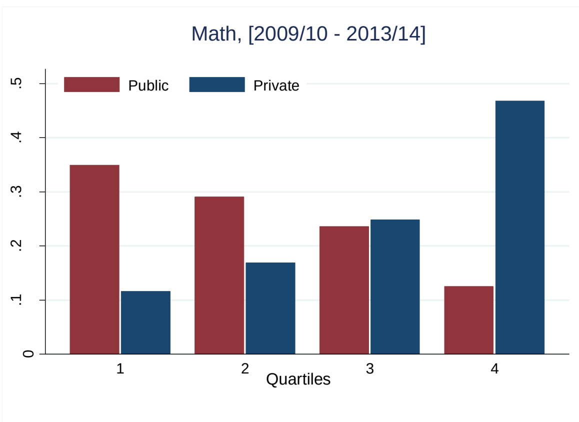
Notes: Y-axis shows estimated transition probabilities. The dotted lines represent 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



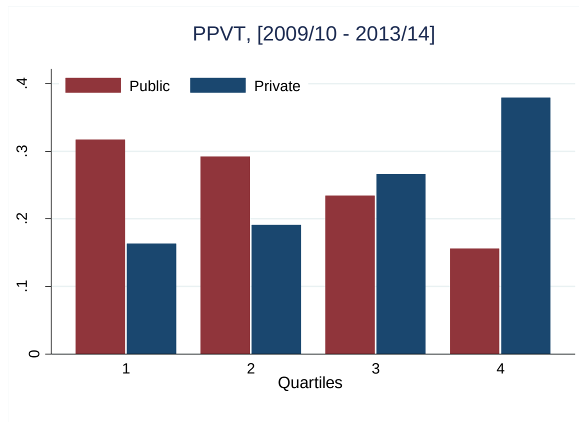


Figure 8. Markov Chain Steady State Distributions

Table 1. Summary Statistics

		Full Sample		Public School		Private School	
		Mean	SD	Mean	SD	Mean	SD
(A) 2006/07 Wave	Math test Score	0.094	0.978	-0.022	0.949	0.451	0.980
	PPVT test score	0.130	0.896	0.004	0.844	0.516	0.942
	Wealth Index	0.427	0.185	0.372	0.153	0.596	0.174
	Age of Child	5.009	0.107	5.003	0.083	5.026	0.160
	Grade Completion by the child	0.299	0.501	0.273	0.500	0.382	0.498
	Male	0.531	0.499	0.518	0.500	0.571	0.496
	Ethnicity (Schedule Caste)	0.207	0.405	0.233	0.423	0.126	0.332
	Ethnicity (Schedule Tribe)	0.184	0.388	0.206	0.405	0.115	0.320
	Ethnicity (Backward Caste)	0.432	0.496	0.450	0.498	0.377	0.486
	Ethnicity (Other Caste)	0.177	0.382	0.111	0.314	0.382	0.487
	Religion (Hindu)	0.886	0.318	0.898	0.303	0.848	0.360
	Household Size	5.496	2.302	5.537	2.317	5.372	2.258
	Residence (Rural)	0.859	0.349	0.961	0.194	0.545	0.499
	N	778		587		191	
		Mean	SD	Mean	SD	Mean	SD
(B) 2009/10 Wave	Math test Score	0.047	0.943	-0.118	1.037	0.258	0.759
	PPVT test score	0.029	0.959	-0.155	1.022	0.263	0.815
	Wealth Index	0.516	0.178	0.427	0.147	0.630	0.147
	Age of Child	7.834	0.385	7.853	0.371	7.811	0.401
	Grade Completion by the child	1.683	0.967	1.907	0.939	1.397	0.927
	Male	0.529	0.499	0.472	0.499	0.600	0.490
	Ethnicity(Schedule Caste)	0.184	0.388	0.233	0.423	0.123	0.328
	Ethnicity(Schedule Tribe)	0.146	0.353	0.202	0.401	0.075	0.264
	Ethnicity(Backward Caste)	0.467	0.499	0.468	0.499	0.464	0.499
	Ethnicity(Other Caste)	0.203	0.402	0.097	0.296	0.338	0.473
	Religion(Hindu)	0.872	0.334	0.903	0.296	0.833	0.373
	Household Size	5.457	2.296	5.529	2.291	5.365	2.301
	Residence (Rural)	0.742	0.438	0.920	0.271	0.514	0.500
	N	1839		1031		808	
		Mean	SD	Mean	SD	Mean	SD
(C) 2013/14 Wave	Math test Score	0.071	0.901	-0.129	0.935	0.367	0.757
	PPVT test score	0.046	0.934	-0.117	0.848	0.288	1.001
	Wealth Index	0.587	0.165	0.516	0.147	0.692	0.131
	Age of Child	11.838	0.381	11.860	0.366	11.807	0.402
	Grade Completion by the child	5.517	1.230	5.676	1.175	5.280	1.272
	Male	0.540	0.499	0.484	0.500	0.623	0.485
	Ethnicity(Schedule Caste)	0.182	0.386	0.239	0.426	0.098	0.297
	Ethnicity(Schedule Tribe)	0.144	0.351	0.195	0.397	0.068	0.252
	Ethnicity(Backward Caste)	0.468	0.499	0.467	0.499	0.469	0.499
	Ethnicity(Other Caste)	0.206	0.405	0.099	0.299	0.365	0.482
	Religion(Hindu)	0.877	0.329	0.894	0.307	0.850	0.357
	Household Size	4.873	1.795	4.871	1.698	4.878	1.930
	Residence (Rural)	0.718	0.450	0.894	0.309	0.457	0.498
	N	1825		1090		735	

Notes: Math test score: IRT Math score; PPVT test score: IRT PPVT score; N = Number of observations. SD = Standard deviation. N in each panel correspond to the number of students enrolled in a type of school in each wave with no missing value for the covariates.

Table 2. Conditional Mobility Gaps, Math

	2006/07-2009-10				2009/10-2013/14			
	Upward		Downward		Upward		Downward	
	δ=0.00	δ=0.10	δ=0.00	δ=0.10	δ=0.00	δ=0.10	δ=0.00	δ=0.10
Public School	0.174***	0.187***	-0.189***	-0.171***	-0.151***	-0.135***	0.150***	0.107***
	(0.060)	(0.055)	(0.060)	(0.059)	(0.035)	(0.034)	(0.035)	(0.032)
Household size	0.005	-0.005	-0.005	0.002	0.004	0.004	-0.004	0.004
	(0.007)	(0.008)	(0.007)	(0.007)	(0.005)	(0.005)	(0.005)	(0.005)
Ethnicity (Schedule Caste)	0.288***	0.273***	-0.293***	-0.308***	-0.158***	-0.124***	0.158***	0.162***
	(0.061)	(0.057)	(0.061)	(0.060)	(0.044)	(0.043)	(0.044)	(0.040)
Ethnicity (Backward Caste)	0.233***	0.218***	-0.236***	-0.213***	-0.135***	-0.096**	0.132***	0.120***
	(0.054)	(0.048)	(0.053)	(0.055)	(0.038)	(0.038)	(0.038)	(0.034)
Ethnicity (Other Caste)	0.306***	0.282***	-0.299***	-0.282***	-0.149***	-0.143***	0.147***	0.141***
	(0.067)	-0.066	(0.069)	(0.069)	(0.047)	(0.046)	(0.047)	(0.042)
Religion (Hindu)	0.146**	0.066	-0.132**	-0.134**	0.015	-0.024	-0.016	0.033
	(0.063)	(0.060)	(0.063)	(0.063)	(0.041)	(0.039)	(0.041)	(0.037)
Wealth Index	0.105	0.276**	-0.130	-0.263*	0.003	-0.102	0.006	-0.182**
	(0.140)	(0.134)	(0.139)	(0.136)	(0.098)	(0.094)	(0.098)	(0.088)
Rural	0.065	0.086	-0.059	-0.080	0.017	0.039	-0.016	-0.064*
	(0.070)	(0.068)	(0.070)	(0.069)	(0.037)	(0.035)	(0.037)	(0.033)
Age	0.176	0.030	-0.186	-0.053	-0.121***	-0.055*	0.121***	0.074**
	(0.149)	(0.156)	(0.150)	(0.156)	(0.033)	(0.032)	(0.033)	(0.030)
Male	0.003	-0.001	-0.010	-0.040	-0.005	-0.024	0.004	0.018
	(0.039)	-0.038	(0.039)	(0.038)	(0.026)	(0.024)	(0.026)	(0.024)
Constant	-0.971	-0.332	2.037***	1.288	1.599***	0.939***	-0.598**	-0.302
	(0.766)	(0.804)	(0.771)	(0.797)	(0.271)	(0.263)	(0.271)	(0.247)
Observations	630	630	630	630	1,553	1,553	1,553	1,553
R-squared	0.073	0.070	0.074	0.077	0.035	0.020	0.035	0.030

Notes: Estimation by OLS. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 3. Conditional Mobility Gaps, PPVT

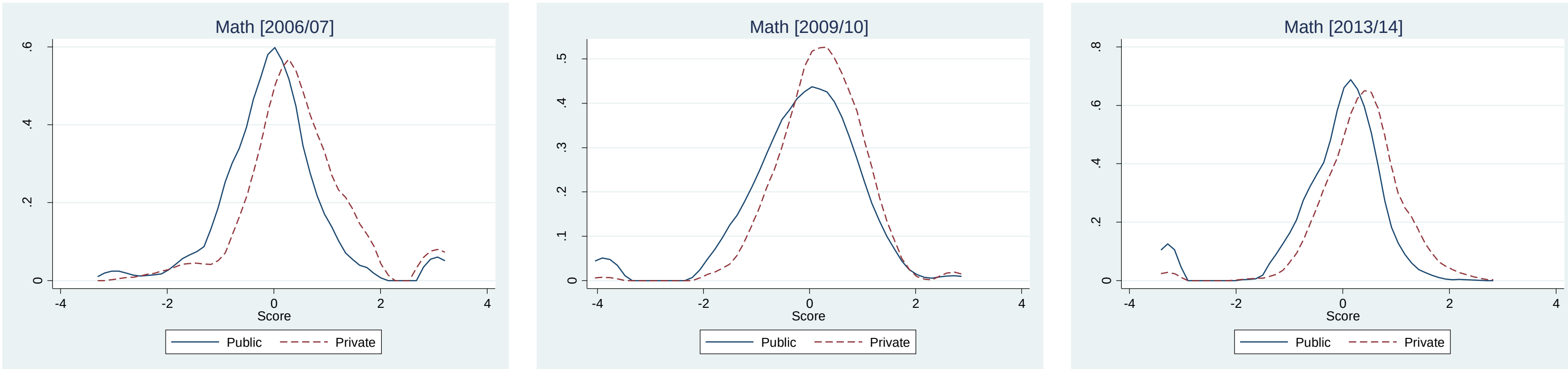
	2006/07-2009-10				2009/10-2013/14			
	Upward		Downward		Upward		Downward	
	δ=0.00	δ=0.10	δ=0.00	δ=0.10	δ=0.00	δ=0.10	δ=0.00	δ=0.10
Public School	0.099*	0.118**	-0.100*	-0.004	-0.068**	-0.093***	0.066*	0.018
	(0.060)	(0.059)	(0.060)	(0.057)	(0.035)	(0.034)	(0.035)	(0.032)
Household size	-0.003	-0.003	0.003	0.009	0.000	0.008	0.000	0.001
	(0.008)	(0.008)	(0.008)	(0.008)	(0.006)	(0.005)	(0.006)	(0.005)
Ethnicity (Schedule Caste)	0.312***	0.257***	-0.304***	-0.264***	0.056	0.029	-0.056	-0.034
	(0.060)	(0.056)	(0.060)	(0.062)	(0.045)	(0.043)	(0.045)	(0.043)
Ethnicity (Backward Caste)	0.300***	0.228***	-0.292***	-0.250***	0.0278	-0.018	-0.031	-0.017
	(0.052)	(0.047)	(0.053)	(0.055)	(0.039)	(0.038)	(0.039)	(0.038)
Ethnicity (Other Caste)	0.444***	0.351***	-0.437***	-0.349***	0.036	-0.028	-0.039	-0.046
	(0.067)	(0.065)	(0.067)	(0.068)	(0.048)	(0.046)	(0.048)	(0.045)
Religion (Hindu)	0.081	0.038	-0.083	-0.057	-0.022	-0.052	0.034	0.004
	(0.060)	(0.059)	(0.060)	(0.059)	(0.041)	(0.039)	(0.041)	(0.039)
Wealth Index	-0.122	-0.040	0.122	-0.076	0.140	0.025	-0.140	-0.093
	(0.142)	(0.130)	(0.141)	(0.136)	(0.099)	(0.093)	(0.099)	(0.093)
Rural	0.159**	0.105	-0.159**	-0.185***	0.150***	0.098***	-0.153***	-0.093***
	(0.068)	(0.067)	(0.068)	(0.066)	(0.037)	(0.035)	(0.037)	(0.035)
Age	0.285**	0.289*	-0.286**	-0.118	-0.021	0.007	0.015	-0.006
	(0.140)	(0.150)	(0.140)	(0.146)	(0.033)	(0.031)	(0.033)	(0.031)
Male	0.031	0.037	-0.028	-0.052	-0.045*	-0.046*	0.045*	0.038
	(0.039)	(0.038)	(0.039)	(0.038)	(0.026)	(0.024)	(0.026)	(0.024)
Constant	-1.420**	-1.499*	2.416***	1.402*	0.395	0.190	0.644**	0.569**
	(0.719)	(0.766)	(0.719)	(0.745)	(0.275)	(0.259)	(0.275)	(0.259)
Observations	630	630	630	630	1,553	1,553	1,553	1,553
R-squared	0.092	0.067	0.090	0.068	0.014	0.013	0.014	0.007

Notes: Estimation by OLS. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Appendix

Measuring the Dynamics of the Achievement Gap Between Public and Private School Students During Early Life in India

Panel A: Math Test Scores



Panel B: PPVT Test Scores

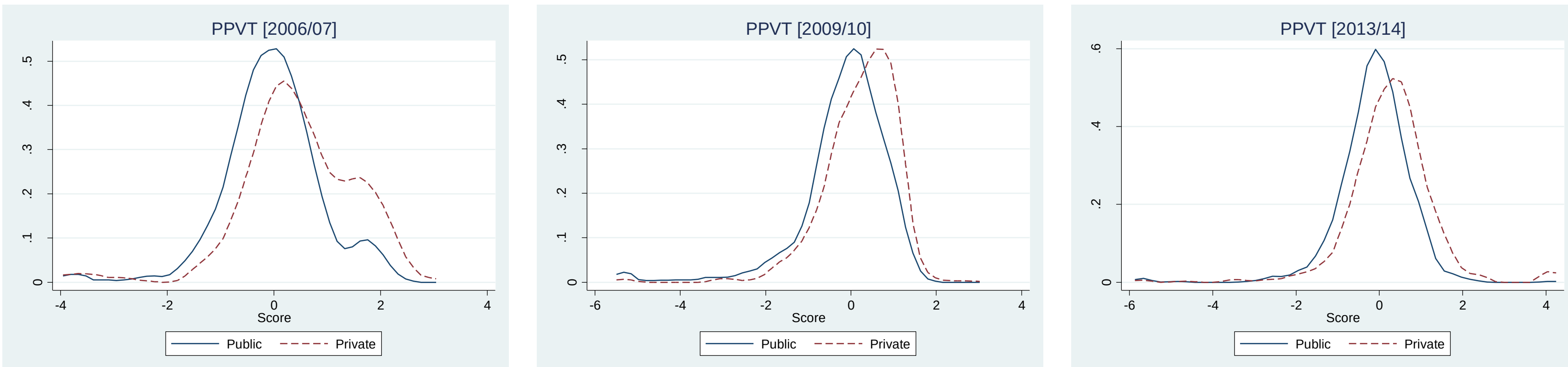


Figure A1. Kernel Density Plots of Test Scores in Math and PPVT by School Type

Table A1. Transition probability estimates by school type, Math

Panel A: 2006/07 - 2009/10	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.426	0.316	0.147	0.110	0.286	0.381	0.190	0.143	0.141	-0.065	-0.043	-0.033
[N_{PU} = 136,	(0.057)	(0.059)	(0.050)	(0.042)	(0.090)	(0.087)	(0.089)	(0.073)	(0.114)	(0.104)	(0.105)	(0.084)
N_{PR} = 21]												
Q2	0.289	0.250	0.266	0.195	0.286	0.250	0.357	0.107	0.003	0.000	-0.092	0.088
[N_{PU} = 128,	(0.063)	(0.064)	(0.058)	(0.051)	(0.087)	(0.077)	(0.089)	(0.056)	(0.122)	(0.107)	(0.115)	(0.080)
N_{PR} = 28]												
Q3	0.209	0.227	0.327	0.236	0.109	0.174	0.370	0.348	0.100	0.053	-0.042	-0.111
[N_{PU} = 110,	(0.064)	(0.078)	(0.076)	(0.074)	(0.048)	(0.071)	(0.071)	(0.069)	(0.084)	(0.114)	(0.104)	(0.110)
N_{PR} = 46]												
Q4	0.136	0.227	0.239	0.398	0.132	0.176	0.265	0.426	0.004	0.051	-0.026	-0.029
[N_{PU} = 88,	(0.072)	(0.078)	(0.084)	(0.094)	(0.044)	(0.057)	(0.056)	(0.068)	(0.088)	(0.097)	(0.105)	(0.129)
N_{PR} = 68]												
Panel B: 2009/10 - 2013/14	**Public**				**Private**				**Public - Private**			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.618	0.246	0.113	0.024	0.500	0.340	0.110	0.050	0.118*	-0.094*	0.003	-0.026
[N_{PU} = 293,	(0.028)	(0.026)	(0.021)	(0.009)	(0.049)	(0.044)	(0.032)	(0.024)	(0.061)	(0.055)	(0.041)	(0.026)
N_{PR} = 100]												
Q2	0.270	0.354	0.288	0.089	0.159	0.318	0.312	0.212	0.111**	0.036	-0.024	-0.123***
[N_{PU} = 226,	(0.034)	(0.035)	(0.035)	(0.022)	(0.031)	(0.034)	(0.034)	(0.029)	(0.049)	(0.051)	(0.053)	(0.036)
N_{PR} = 170]												
Q3	0.194	0.316	0.296	0.194	0.097	0.161	0.344	0.398	0.097**	0.154***	-0.048	-0.204***
[N_{PU} = 206,	(0.033)	(0.036)	(0.038)	(0.031)	(0.019)	(0.026)	(0.032)	(0.030)	(0.040)	(0.046)	(0.052)	(0.048)
N_{PR} = 186]												
Q4	0.072	0.227	0.343	0.359	0.014	0.077	0.208	0.700	0.057**	0.149***	0.135***	-0.341***
[N_{PU} = 181,	(0.022)	(0.037)	(0.041)	(0.037)	(0.010)	(0.021)	(0.027)	(0.029)	(0.025)	(0.044)	(0.051)	(0.050)
N_{PR} = 207]												

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A2. Transition probability estimates by school type, PPVT

Panel A: 2006/07 - 2009/10	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.450	0.271	0.179	0.100	0.368	0.211	0.105	0.316	0.082	0.061	0.073	-0.216**
[N_{PU} = 136, N_{PR} = 21]	(0.063)	(0.058)	(0.046)	(0.039)	(0.096)	(0.081)	(0.085)	(0.080)	(0.130)	(0.108)	(0.098)	(0.091)
Q2	0.295	0.254	0.295	0.156	0.222	0.250	0.361	0.167	0.073	0.004	-0.066	-0.011
[N_{PU} = 122, N_{PR} = 36]	(0.068)	(0.068)	(0.065)	(0.055)	(0.070)	(0.088)	(0.087)	(0.054)	(0.108)	(0.125)	(0.117)	(0.081)
Q3	0.188	0.308	0.239	0.265	0.195	0.195	0.390	0.220	-0.007	0.113	-0.151	0.045
[N_{PU} = 117, N_{PR} = 41]	(0.055)	(0.068)	(0.071)	(0.066)	(0.062)	(0.072)	(0.074)	(0.069)	(0.085)	(0.105)	(0.114)	(0.102)
Q4	0.125	0.250	0.284	0.341	0.057	0.143	0.186	0.614	0.068	0.107	0.098	-0.273**
[N_{PU} = 88, N_{PR} = 68]	(0.074)	(0.095)	(0.093)	(0.095)	(0.035)	(0.043)	(0.057)	(0.055)	(0.085)	(0.106)	(0.113)	(0.115)

Panel B: 2009/10 - 2013/14	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.569	0.281	0.113	0.037	0.319	0.261	0.286	0.134	0.250***	0.021	-0.173***	-0.098***
[N_{PU} = 274, N_{PR} = 119]	(0.030)	(0.027)	(0.022)	(0.013)	(0.041)	(0.042)	(0.039)	(0.031)	(0.053)	(0.051)	(0.045)	(0.035)
Q2	0.287	0.364	0.240	0.109	0.246	0.246	0.261	0.246	0.041	0.118**	-0.021	-0.138***
[N_{PU} = 258, N_{PR} = 134]	(0.030)	(0.032)	(0.029)	(0.023)	(0.038)	(0.038)	(0.034)	(0.037)	(0.054)	(0.057)	(0.046)	(0.047)
Q3	0.157	0.266	0.341	0.236	0.086	0.196	0.331	0.387	0.071*	0.070	0.009	-0.151***
[N_{PU} = 229, N_{PR} = 163]	(0.029)	(0.038)	(0.037)	(0.033)	(0.021)	(0.031)	(0.034)	(0.033)	(0.037)	(0.055)	(0.057)	(0.053)
Q4	0.103	0.221	0.310	0.366	0.109	0.130	0.215	0.547	-0.006	0.091*	0.096	-0.181***
[N_{PU} = 145, N_{PR} = 247]	(0.033)	(0.043)	(0.051)	(0.050)	(0.023)	(0.023)	(0.026)	(0.031)	(0.046)	(0.052)	(0.062)	(0.067)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A3. Upward rank mobility estimates by school type, Math

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Panel A: 2006/07 - 2009/10									
Q1	0.772	0.762	0.010	0.728	0.714	0.014	0.647	0.714	-0.067
[N_{PU} = 136, N_{PR} = 21]	(0.053)	(0.079)	(0.104)	(0.057)	(0.085)	(0.113)	(0.060)	(0.087)	(0.117)
Q2	0.578	0.500	0.078	0.531	0.500	0.031	0.453	0.393	0.060
[N_{PU} = 128, N_{PR} = 28]	(0.069)	(0.098)	(0.131)	(0.067)	(0.096)	(0.128)	(0.070)	(0.091)	(0.123)
Q3	0.445	0.522	-0.076	0.373	0.478	-0.106	0.300	0.391	-0.091
[N_{PU} = 110, N_{PR} = 46]	(0.084)	(0.072)	(0.118)	(0.084)	(0.069)	(0.118)	(0.082)	(0.071)	(0.115)
Q4	0.227	0.162	0.066	0.216	0.103	0.113	0.100	0.053	0.047
[N_{PU} = 88, N_{PR} = 68]	(0.088)	(0.054)	(0.106)	(0.098)	(0.043)	(0.111)	(0.092)	(0.051)	(0.106)
Panel B: 2009/10 - 2013/14									
	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.683	0.810	-0.127**	0.509	0.600	-0.091	0.451	0.520	-0.069
[N_{PU} = 293, N_{PR} = 100]	(0.030)	(0.041)	(0.051)	(0.031)	(0.051)	(0.063)	(0.031)	(0.051)	(0.063)
Q2	0.562	0.659	-0.097*	0.478	0.612	-0.134**	0.412	0.524	-0.112*
[N_{PU} = 226, N_{PR} = 170]	(0.039)	(0.036)	(0.058)	(0.039)	(0.037)	(0.056)	(0.035)	(0.041)	(0.059)
Q3	0.311	0.597	-0.286***	0.262	0.516	-0.254***	0.223	0.435	-0.212***
[N_{PU} = 206, N_{PR} = 186]	(0.036)	(0.034)	(0.054)	(0.037)	(0.036)	(0.056)	(0.033)	(0.034)	(0.052)
Q4	0.116	0.372	-0.256***	0.071	0.243	-0.172***	0.048	0.189	-0.141***
[N_{PU} = 181, N_{PR} = 207]	(0.026)	(0.032)	(0.043)	(0.025)	(0.031)	(0.042)	(0.022)	(0.032)	(0.042)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A4. Upward rank mobility estimates by school type, PPVT

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Panel A: 2006/07 - 2009/10									
Q1	0.779	0.737	0.042	0.679	0.684	-0.006	0.629	0.579	0.050
[N_{PU} = 136, N_{PR} = 21]	(0.055)	(0.091)	(0.115)	(0.057)	(0.095)	(0.121)	(0.061)	(0.098)	(0.127)
Q2	0.582	0.639	-0.057	0.508	0.583	-0.075	0.459	0.500	-0.041
[N_{PU} = 122, N_{PR} = 36]	(0.071)	(0.079)	(0.114)	(0.070)	(0.082)	(0.115)	(0.067)	(0.085)	(0.118)
Q3	0.393	0.366	0.027	0.325	0.293	0.032	0.265	0.268	-0.003
[N_{PU} = 117, N_{PR} = 41]	(0.075)	(0.087)	(0.120)	(0.070)	(0.077)	(0.106)	(0.069)	(0.070)	(0.103)
Q4	0.250	0.257	-0.007	0.135	0.264	-0.129	0.068	0.194	-0.127
[N_{PU} = 88, N_{PR} = 68]	(0.087)	(0.050)	(0.105)	(0.085)	(0.053)	(0.103)	(0.075)	(0.060)	(0.097)
Panel B: 2009/10 - 2013/14									
	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.755	0.849	-0.093**	0.569	0.782	-0.212***	0.478	0.740	-0.261***
[N_{PU} = 274, N_{PR} = 119]	(0.029)	(0.033)	(0.047)	(0.030)	(0.038)	(0.051)	(0.030)	(0.041)	(0.053)
Q2	0.527	0.642	-0.115**	0.453	0.590	-0.136**	0.376	0.537	-0.161***
[N_{PU} = 258, N_{PR} = 134]	(0.036)	(0.041)	(0.058)	(0.035)	(0.041)	(0.056)	(0.034)	(0.041)	(0.057)
Q3	0.402	0.595	-0.193***	0.323	0.454	-0.131**	0.266	0.380	-0.114**
[N_{PU} = 229, N_{PR} = 163]	(0.040)	(0.037)	(0.059)	(0.038)	(0.041)	(0.058)	(0.034)	(0.034)	(0.052)
Q4	0.179	0.279	-0.100*	0.153	0.209	-0.057	0.077	0.117	-0.040
[N_{PU} = 145, N_{PR} = 247]	(0.041)	(0.026)	(0.053)	(0.038)	(0.029)	(0.051)	(0.037)	(0.031)	(0.047)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A5. Downward rank mobility estimates by school type, Math

Panel A: 2006/07 - 2009/10

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.228	0.238	-0.010	0.202	0.188	0.014	0.185	0.154	0.031
[N_{PU} = 136, N_{PR} = 21]	(0.053)	(0.077)	(0.102)	(0.057)	(0.094)	(0.119)	(0.064)	(0.077)	(0.101)
Q2	0.422	0.500	-0.078	0.375	0.500	-0.125	0.313	0.393	-0.080
[N_{PU} = 128, N_{PR} = 28]	(0.069)	(0.100)	(0.133)	(0.068)	(0.094)	(0.130)	(0.064)	(0.093)	(0.127)
Q3	0.555	0.478	0.076	0.500	0.413	0.087	0.473	0.326	0.147
[N_{PU} = 110, N_{PR} = 46]	(0.084)	(0.073)	(0.119)	(0.085)	(0.070)	(0.119)	(0.080)	(0.071)	(0.112)
Q4	0.773	0.838	-0.066	0.682	0.735	-0.053	0.636	0.662	-0.025
[N_{PU} = 88, N_{PR} = 68]	(0.089)	(0.054)	(0.108)	(0.095)	(0.059)	(0.119)	(0.097)	(0.061)	(0.122)

Panel B: 2009/10 - 2013/14

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.317	0.190	0.127**	0.214	0.090	0.124**	0.116	0.042	0.074*
[N_{PU} = 293, N_{PR} = 100]	(0.029)	(0.040)	(0.051)	(0.032)	(0.039)	(0.050)	(0.031)	(0.026)	(0.042)
Q2	0.438	0.341	0.097*	0.385	0.276	0.108*	0.314	0.206	0.108**
[N_{PU} = 226, N_{PR} = 170]	(0.039)	(0.036)	(0.058)	(0.039)	(0.034)	(0.056)	(0.039)	(0.031)	(0.053)
Q3	0.684	0.403	0.281***	0.592	0.376	0.216***	0.510	0.285	0.225***
[N_{PU} = 206, N_{PR} = 186]	(0.036)	(0.034)	(0.053)	(0.040)	(0.033)	(0.055)	(0.041)	(0.030)	(0.054)
Q4	0.884	0.623	0.261***	0.779	0.473	0.306***	0.696	0.348	0.348***
[N_{PU} = 181, N_{PR} = 207]	(0.026)	(0.032)	(0.043)	(0.033)	(0.031)	(0.047)	(0.036)	(0.031)	(0.050)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A6. Downward rank mobility estimates by school type, PPVT

Panel A: 2006/07 - 2009/10									
	$\delta = 0.00$			$\delta = 0.05$			$\delta = 0.10$		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.214	0.263	-0.049	0.196	0.067	0.130	0.172	0.125	0.047
[N_{PU} = 136, N_{PR} = 21]	(0.053)	(0.093)	(0.116)	(0.052)	(0.092)	(0.119)	(0.060)	(0.114)	(0.148)
Q2	0.410	0.361	0.049	0.369	0.306	0.063	0.303	0.250	0.053
[N_{PU} = 122, N_{PR} = 36]	(0.070)	(0.078)	(0.113)	(0.073)	(0.073)	(0.112)	(0.070)	(0.075)	(0.112)
Q3	0.607	0.634	-0.027	0.564	0.488	0.076	0.538	0.439	0.099
[N_{PU} = 117, N_{PR} = 41]	0.075	(0.088)	(0.122)	(0.076)	(0.082)	(0.117)	(0.075)	(0.078)	(0.115)
Q4	0.750	0.743	0.007	0.739	0.614	0.124	0.659	0.471	0.188
[N_{PU} = 88, N_{PR} = 68]	(0.087)	(0.050)	(0.105)	(0.089)	(0.060)	(0.113)	(0.093)	(0.059)	(0.115)

Panel B: 2009/10 - 2013/14									
	$\delta = 0.00$			$\delta = 0.05$			$\delta = 0.10$		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.241	0.151	0.090*	0.175	0.107	0.069	0.098	0.096	0.002
[N_{PU} = 274, N_{PR} = 119]	(0.029)	(0.033)	(0.047)	(0.029)	(0.032)	(0.046)	(0.027)	(0.037)	(0.047)
Q2	0.473	0.358	0.115**	0.395	0.291	0.104*	0.333	0.231	0.102**
[N_{PU} = 258, N_{PR} = 134]	(0.036)	(0.041)	(0.058)	(0.033)	(0.040)	(0.055)	(0.032)	(0.039)	(0.050)
Q3	0.598	0.405	0.193***	0.550	0.362	0.188***	0.472	0.331	0.140**
[N_{PU} = 229, N_{PR} = 163]	(0.040)	(0.037)	(0.059)	(0.041)	(0.036)	(0.060)	(0.041)	(0.035)	(0.058)
Q4	0.814	0.721	0.093*	0.752	0.611	0.140**	0.614	0.518	0.096
[N_{PU} = 145, N_{PR} = 247]	(0.041)	(0.027)	(0.054)	(0.046)	(0.031)	(0.061)	(0.050)	(0.031)	(0.066)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.