Is Human-Capital Acquisition Optimizing Behaviour? A Life-cycle Analysis

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WP/04/016/ECO
March 2016

Noname manuscript No.
(will be inserted by the editor)

Is Human-Capital Acquisition Optimizing Behaviour? A Life-cycle Analysis

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Received: date / Accepted: date

Abstract This study develops a mathematical model to replicate the empirical findings of child-labour and differential levels of human-capital acquisition for different income groups. The study explores income groups with different levels of financial access and human-capital investment options and examines a two-generation, seventy periods' life-cycle problem of an altruistic parent valuing the human-capital of his offspring and facing competing challenges of his consumption needs during working life and retirement. The problem is solved using dynamic programming where life-cycle profiles are generated using simulations. This study contributes by integrating the empirical literature on child-labour with the literature on intergenerational set-up and risk.

Keywords Child labour · Human capital
JEL classification: D60; I26;

1 Literature Review

The earliest works linking human capital accumulation to life-cycle earnings were done by Mincer (1958), Becker (1964) and Ben-Porath (1967). Education is one field where the need for policy intervention through freely provided public education, vouchers, subsidies is taken for granted. The main argument often is the existence of credit market imperfections. A considerable literature exists about the asymmetric information and moral hazard problem which can lead to equilibrium credit rationing (Stiglitz & Weiss (1981)). In the absence of credit markets the child is dependent on his parents for his educational needs. Hence, the intra-family relations along with binding liquidity constraints become important for evaluation of human capital investments. A strong correlation between one's educational attainment and one's parental education is documented extensively in the literature. A part of the reason is attributed to heritability of traits i.e children inherit ability, personality and preferences of parents. The other reason is human capital acquisition i.e educated parents due to their preferences or higher wealth may invest more in their children's education.

Inputs by family along with other environmental conditions play a role in shaping a child. Todd & Wolpin (2003) have estimated a dynamic child outcome production function viewing child outcome as a cumulative process with educational inputs and heritable traits influencing the the final quality. Similarly Cunha & Heckman (2008), Cunha et al. (2010) have also estimated, in a model of skill formation, the child cognitive and non cognitive outcomes in a dynamic factor model. Similarly, Keane & Wolpin (2001), Johnson (2010) model the differences in parental transfers in explaining schooling outcomes.

Human capital investment may take several forms like parental time, investing in goods complementary to learning or direct subsidies like tuition fee payments. In the given scenario, binding borrowing constraints on the family may thus

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influence the human capital attainment of the child. The link between borrowing constraints and under-investment in education is well documented in literature (G. S. Becker (1960), Schultz (1961), Friedman (2002)). Friedman noted that the physical capital analogy in education would be buying a share in a person's future earnings. However, due to moral hazard problems a market cannot develop. Hence, borrowing constraints remain a reality and continue to affect investment. More recently, Ellwood et al. (2000), Carneiro & Heckman (2002), Belley & Lochner (2007), Lochner & Monge-Naranjo (2008) have linked lower college enrolment to borrowing constraints in students from low income families. Lochner & Monge-Naranjo (2008) work with a 3-period model of human capital formation and show that the empirical findings of positive correlation between educational attainment and family income is consistent with the impact of binding borrowing constraints. However, they do not model uncertainty in labour market.

Del Boca et al. (2010) estimate a model of cognitive development of the child incorporating the endogenous dynamic relationships between child quality and parental employment (also mother's). Lee & Seshadri (2012), along the lines of G. Becker & Tomes (1994) model human capital investment within and across generations under incomplete markets. Their main aim is to disentangle exogenous ability transmission with endogenous transmission mechanisms like borrowing constraints and parental human capital. They conclude that both the mechanisms have significant impact.

Lochner & Caucutt (2012) investigate the role of family borrowing constraints in deciding human capital investments in children in the early and late stages. They assume imperfect credit markets and an intergenerational setup. A key finding is that a significant chunk of parents are borrowing constrained.

In addition to the above mentioned literature, there are several studies which investigate human-capital accumulation, taxation, subsidies and growth using large-scale OLG models in a general equilibrium framework. In a model of endogenous human-capital, Krueger & Ludwig (2013) investigate the optimal mix of taxation and education subsidies from a welfare maximizing fiscal policy perspective. Similarly, Ludwig et al. (2012) predict the welfare losses, in an environment of endogenous human-capital, of an ageing demography. Several other studies like Benabou (2002) and Bohacek & Kapicka (2008) have focused on human-capital acquisition, optimal taxation policies and education subsidies.

While most of the inter-generational literature looks at human-capital acquisition under different scenarios of heritability of traits, parental investments and borrowing constraints, the impact of uncertainty in wages and financial assets on life-cycle variables including human-capital acquisition has largely remained unexplored. The next section reviews the finance literature dealing with asset risk, income risk, portfolio choice and borrowing constraints.

2 A Two-Generation, Discrete Time, Life-Cycle Model

2.1 The Environment

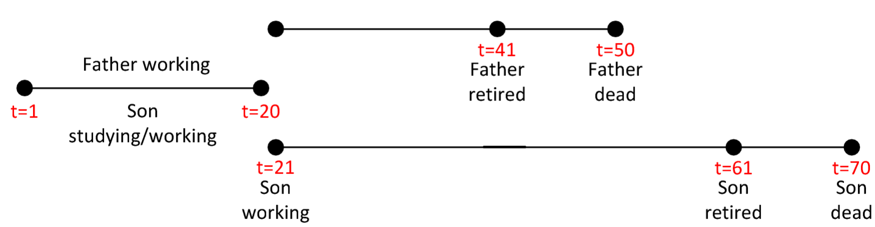


Fig. 1: Time-line : Two-generation, seventy discrete time-periods' life-cycle

The model assumes a single parent (father) with a single offspring (son) both of whom would be jointly referred to as the household.

Time is discrete and the life-cycle model starts from time, $t = 1$, when the son is just an infant (age=1) and the father has started working (age=21).
"t" denotes absolute time in years, "age" denotes the age in years of the person in focus (either father or son). For the son, t and age will be the same. There are 70 time periods in the model.
A phase is defined as the son's phase in life: education phase and adult phase. The first 20 time periods ($1 \leq t \leq 20$) have been defined as the education phase. Adult phase starts from $t = 21$ onwards. The education phase is also when the father and son are a joint household. Hence, the terms education phase and joint phase are used interchangeably.

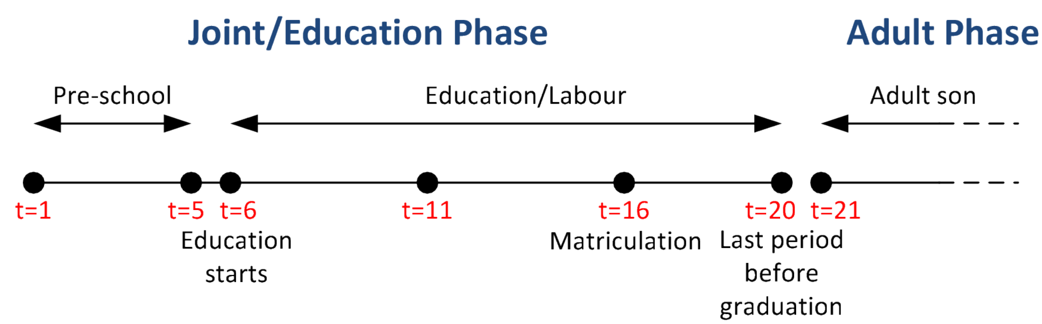


Fig. 2: Time-line: Important points in the son's education phase

In the first five years (pre-school stage) there are neither education nor labour options available to the son. The only expense the father incurs on the son, during this period, is consumption expense. Even though a child consumes less than the parent, in the simulations the father-son consumption expenditures are assumed as equal since health related expenses of the child are not explicitly modelled.

From $6 \leq age < 20$ the son can either go to acquire human-capital or work in the informal child-labour market to augment the family income. (Note: A definition of a child is someone who is not an adult). The time of entry of the independent (grown-up) son into the formal labour market is denoted as $t = sw = 21$ (son working) which is also the time the son separates and starts a new family.

From $t = sw = 21$ onwards, the father and son are economically separate and have no interaction except for a last period (of father's life) bequest. At time $t = fr = 41$ the father retires (father's age=61) and the father is dead at time $t = fd = 50$ (father's age=70). He leaves a pre-determined bequest, which is a constant fraction of his last-period wealth, to his son.

The son starts his adult life with zero assets. The son retires at $t = sr = 61$ and dies at $t = sd = 70$ which is also the last period of the model.

Human-capital
From, $6 \leq t \leq 20$, the father has an option of either educating the son or sending him to the child-labour market or a

combination of both. The son is endowed with a unit amount of time in each period. The fraction of time devoted to education in time-period t is denoted as λ_t. The father also decides the amount of books b_t to be invested in each period. The son's future income as an adult depends on the level of human capital accumulated in the education phase ($6 \leq t \leq 20$).

The household faces borrowing constraints every period as the father cannot borrow against his son's future labour income due to moral hazard problems. The model also assumes that the father has no borrowing options against his own future labour income due to labour-income risk.

The father's human capital $H_{f,t} = H_{dad}$ is assumed constant over time. Subscript f stands for father, s stands for son.

The Ben-Porath (1967) model was used for the human capital production function. The child's human capital at period $t+1$, in the education phase, is the function of previous period's human capital $H_{s,t}$, ability level θ, time fraction spent studying λ_t, books invested b_t and the rate of depreciation δ.

$$\begin{aligned} H_{f,t+1} &= H_{f,t} = H_{dad} \\ H_{s,t+1} &= f(\theta, \lambda_t, H_{s,t}, b_t) \\ &= H_{s,t}(1-\delta) + \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} \end{aligned} \tag{1}$$

The human capital of the pre-school son is assumed to stay constant and is assumed as equal to the base level of human capital $\bar{H}$(which is the lowest possible level of human-capital and there is no further depreciation from that). Human-capital acquisition can start from age 6 onwards.
Since, the nutritional effect on human-capital is not modelled in the pre-school stage, human-capital is assumed to remain at the base level upto age 6. Also, since the father is solving the life-cycle model at $t = 1$ and assumes the son will be a separate household once he reaches age 21, the father only takes into account formal education upto graduation. Human-capital accumulation prospects of the independent son through On-the-Job Training (OJT) is not modelled.

The maximum $H_{s,t+1}$ the child can have corresponds to $\lambda_t = \lambda_{max}$ and $b_t = b_{max}$.

$$H_{s,t+1}(max) = f(\theta, \lambda_{max}, H_{s,t}, b_{max}) = H_{s,t}(1-\delta) + \theta(\lambda_{max})^{aa}(H_{s,t})^{bb}(b_{max})^{cc} \tag{2}$$

$\lambda_{max} = 0$ when the child is in the pre-school stage and equal to 1 for other periods in the education phase.
b_{max} is the maximum investment possible in books. b_{max} is zero at pre-school and then constant for a given income category. It varies for different income categories to reflect the inaccessibility of good schools for poor households.

The father has a per-period dis-utility from not being able to afford the maximum inputs (time and books) for his child. The dis-utility is proportional to the amount of human capital foregone, $\Big(f(\theta, \lambda_{max}, H_{s,t}, b_{max}) - f(\theta, \lambda_t, H_{s,t}, b_t)\Big)$.

The amount of dis-utility to a father due to lower inputs, $\lambda_t < 1$ and $b_t < b_{max}$ is :

$$pen.\Big(f(\theta, \lambda_{max}, H_{s,t}, b_{max}) - f(\theta, \lambda_t, H_{s,t}, b_t)\Big) \tag{3}$$

where *pen* is the amount of dis-utility per-unit of human capital foregone.

Father's wage
Wages are subjected to a per period wage shock e_t which can take two values, $e_t = \{e_{low}, e_{high}\}$ such that $\mathbf{E}[e_t] = 1$. The actual wage of an adult with human-capital level H_{dad} at time t for given wage shock realization e_t

$$WAGE(H_{dad}, t, e_t) = \bar{G}^{t-1} H_{dad} e_t \tag{4}$$

where $\bar{G}$ is the exogenous growth in expected wage rate every year.

Therefore, the minimum adult expected wage corresponds to that of an adult with base level of human-capital ($\bar{H}$) and is equal to one at $t = 1$ [1]. In monetary terms 1 would be equivalent to Rs 36,000 per year (Rs 100/day).

[1] it grows exogenously at growth rate $\bar{G}$

Child-labour wages
As mentioned previously, a child is defined as someone who is not a adult. The child-labour wage of the son is a fraction of the minimum adult wage and is assumed to be not a function of the actual human-capital of the son.
At any given time t, for human-capital of son $H_{s,t}$, the child-labour wage per unit-time (CLWAGE) is equal to :

$$CLWAGE(H_{s,t},t,e_t) = \bar{G}^{t-1}H_{relv} \tag{5}$$

where H_{relv} is the relevant, age dependent, child-labour wage rate,
$H_{relv} = 0$ for $0 \leq age \leq 5$,
$H_{relv} = 0.33\bar{H}$ for $6 \leq age \leq 10$,
$H_{relv} = 0.5\bar{H}$ for $11 \leq age \leq 15$,
$\{H_{relv} = H_{matric}$ if $H_{s,t} > H_{matric}$ else
$H_{relv} = \bar{H}$ for $16 \leq age \leq 20\}$

H_{matric} is the matriculation level of human-capital and $\bar{H}$ is the base level of human-capital.
A child investing λ_t fraction of time in education would earn $\bar{G}^{t-1}H_{relv}(1-\lambda_t)$.

For ease of computation, a simplifying assumption is made that the child's labour-wage is not subject to wage shocks. Since, the child-labour wage is anyway a fraction of the minimum adult wage the assumption would not result in major discrepancies in net-household income. Also, since child-labour is a non-market interaction, it need not be completely subjected to labour-market shocks and is anyway not priced as a function of actual human-capital.

Financial assets
Financial assets consist of a risk-less one-period bond of gross real return R_0 and a risky one-period stock with gross real return R_{γ_t} subject to a per period stock shock.

$$R_{\gamma_t} = R_0 + \mu + \gamma_t \tag{6}$$

γ_t is the time t shock to the stock return which can take two values. $\gamma_t = \{\gamma_{low}, \gamma_{high}\}$ such that $\mathbf{E}[\gamma_t] = 0$. μ is the excess return of stock over bond. Shocks to labour-wage e_t and stock returns γ_t can be correlated.

Based on the father's level of human capital, he is assumed to either have no financial access, only bond market access or full financial access. A father with no financial access would have negative returns on savings whereas a father with only bond access will not have the option of investing in stock. Only a father with full financial access would have a savings portfolio consisting of a bond and a stock. Since solving the life-cycle portfolio choice of the highest income category is one of the objectives of this study, a shorter five-year-plans version of the seventy period life-cycle can not be employed for faster computation.

Expected Utility Maximization
Let $\Omega_t = (\gamma_t, e_t)$ denote the vector of the wage shock e_t and stock market shock γ_t at any time t. Both e_t and γ_t are individually uncorrelated over time. However, they can be cross-sectionally correlated. Since, both wage and stock shock can take 2 states each, Ω_t can take 4 possible states $\Big((\gamma_{low}, e_{low}), (\gamma_{low}, e_{high}), (\gamma_{high}, e_{low}), (\gamma_{high}, e_{high})\Big)$.

$\Omega^{(t)} = \Omega_1, \Omega_2, \Omega_3....\Omega_t$ denotes the history of joint shocks upto upto time t. It must be noted that $\Omega^{(t)}$ is different from Ω_t (which denotes the realization of the joint shock at time t). An example of $\Omega^{(t)}$ for t=2 would be $\Omega^{(2)} = \Big((\gamma_{low}, e_{high}), (\gamma_{high}, e_{high})\Big)$.

Utility from consumption has been modelled as a standard CRRA utility,

$$u(C) = C^{1-\xi}/(1-\xi)$$

For resources-in-hand $X_{f,1}$, human-capital of son $H_{s,1}$ and contributed-pension-fund deposit $PF_{f,1}$ at aggregate state Ω_1, the parent seeks to maximize the expected utility from his and son's lifetime consumption accounting for per-period dis-utility for foregone human-capital:

$$\max_{C_{f,t},C_{s,t},C_{g,t},\lambda_t,\alpha_t,b_t}\Bigg(\Big(u(C_{f,1})+\rho u(C_{s,1})\Big)-pen.\Big(f(\theta,\lambda_{max},H_{s,1},b_{max})-f(\theta,\lambda_1,H_{s,1},b_1)\Big)+$$
$$\beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{50}u(C_{f,t}(\Omega^{(t)}))+\rho\Big(\sum_{t=2}^{70}u(C_{s,t}(\Omega^{(t)}))+\sum_{t=21}^{40}u(C_{g,t}(\Omega^{(t)}))\Big)\Big]-$$
$$\beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{20}pen.\Big(f(\theta,\lambda_{max},H_{s,t}(\Omega^{(t)}),b_{max})-f(\theta,\lambda_t(\Omega^{(t)})),H_{s,t}(\Omega^{(t)}),b_t(\Omega^{(t)}))\Big)\Big]\Bigg)$$

$\Omega^{(t)}=\Omega_2,\Omega_3....\Omega_t$ is the history at time t,
$C_{f,t}(\Omega^{(t)})$: father's consumption at time t for history $\Omega^{(t)}$,
$C_{s,t}(\Omega^{(t)})$: son's consumption at time t for history $\Omega^{(t)}$,
$\lambda_t(\Omega^{(t)})$: fraction of education time at time t for history $\Omega^{(t)}$,
$b_t(\Omega^{(t)})$: investment in books at time t for history $\Omega^{(t)}$,
pen : father's dis-utility from a unit of foregone human-capital,
ρ: downward altruism coefficient, is equal to 1 upto $t=20$ and then some constant value lower than one for rest of the time-periods.

The son's future consumption and utility from consumption will depend on his human capital level and the father cares about it through downward altruism. Beyond the utility from consumption that human-capital affords, the father explicitly values the acquisition of human-capital which the model seeks to capture through the penalty term. The model abstracts from a dynastic setup by excluding further generations.

Solution methodology
This study employs Value Function Iteration (Bellman Equations) to solve the above problem where the value associated with each period would be recursively written as a function of that period optimum variables and the value in the subsequent period. The way to solve is using backward induction where the value at the last period is computed first and then iteratively the value at intermediate periods are computed.

Let $V^{(1)}(X_{f,1},H_{s,1},PF_{f,1})$ be the value, after the realization of the joint shock $\Omega_1=(e_1,\gamma_1)$ at $t=1$, of having resources-in-hand $X_{f,1}$[2], human capital level of the son $H_{s,1}$ and contributed-pension-fund deposit $PF_{f,1}$. The terms contributed-pension-fund and pension-fund mean the same in this study and would be used interchangeably. (The 1 superscript of $V^{(1)}$ is not power but just a notation for value at time $t=1$)
Then,

$$V^{(1)}(X_{f,1},H_{s,1},PF_{f,1})=\max_{C_{f,t},C_{s,t},C_{g,t},\lambda_t,\alpha_t,b_t}\Bigg(\Big(u(C_{f,1})+\rho u(C_{s,1})\Big)-$$
$$pen.\Big(f(\theta,\lambda_{max},H_{s,1},b_{max})-f(\theta,\lambda_1,H_{s,1},b_1)\Big)+$$
$$\beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{50}u(C_{f,t}(\Omega^{(t)}))+\rho\Big(\sum_{t=2}^{70}u(C_{s,t}(\Omega^{(t)}))+\sum_{t=21}^{40}u(C_{g,t}(\Omega^{(t)}))\Big)\Big]-$$
$$\beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{20}pen.\Big(f(\theta,\lambda_{max},H_{s,t}(\Omega^{(t)}),b_{max})-f(\theta,\lambda_t(\Omega^{(t)})),H_{s,t}(\Omega^{(t)}),b_t(\Omega^{(t)}))\Big)\Big]\Bigg)$$

($\blacksquare^{(\mathbf{t})}=\Omega_2,\Omega_3,...,\Omega_t$ is history upto time t, $C_{f,t}(\Omega^{(t)})$: father's consumption at time t for history $\Omega^{(t)}$, $C_{s,t}(\Omega^{(t)})$: son's consumption at time t for history $\Omega^{(t)}$, λ_t : fraction of education time at time t for history $\Omega^{(t)}$, b_t : books invested at time t for history $\Omega^{(t)}$, ρ : downward altruism coefficient)

[2] initial asset + father's period 1 contributed-pension-fund deducted wage

Or, substituting optimum values ($*$ denotes optimum)

$$\begin{aligned} V^{(1)}(X_{f,1},H_{s,1},PF_{f,1}) = \Bigg(& \Big(u(C^*_{f,1}) + \rho u(C^*_{s,1})\Big) - \\ & pen.\Big(f(\theta,\lambda_{max},H_{s,1},b_{max}) - f(\theta,\lambda^*_1,H_{s,1},b^*_1)\Big) + \\ & \beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{50} u(C^*_{f,t}(\Omega^{(t)})) + \rho\Big(\sum_{t=2}^{70} u(C^*_{s,t}(\Omega^{(t)})) + \sum_{t=21}^{40} u(C^*_{g,t}(\Omega^{(t)}))\Big)\Big] - \\ & \beta^{t-1}\mathbf{E}\Big[\sum_{t=2}^{20} pen.\Big(f(\theta,\lambda_{max},H_{s,t}(\Omega^{(t)}),b_{max}) - f(\theta,\lambda^*_t(\Omega^{(t)})),H_{s,t}(\Omega^{(t)}),b^*_t(\Omega^{(t)}))\Big)\Big]\Bigg) \end{aligned}$$

Or,

$$\begin{aligned} V^{(1)}(X_{f,1},H_{s,1},PF_{f,1}) = \Bigg(& \Big(u(C^*_{f,1}) + \rho u(C^*_{s,1})\Big) - \\ & pen.\Big(f(\theta,\lambda_{max},H_{s,1},b_{max}) - f(\theta,\lambda^*_1,H_{s,1},b^*_1)\Big) + \\ & \beta^{t-1}\Big[\sum_{t=2}^{50}\sum_{\Omega^{(t)}} \pi(\Omega^{(t)})u(C^*_{f,t}(\Omega^{(t)})) + \\ & \rho\Big(\sum_{t=2}^{70}\sum_{\Omega^{(t)}} \pi(\Omega^{(t)})u(C^*_{s,t}(\Omega^{(t)})) + \sum_{t=21}^{40}\sum_{\Omega^{(t)}} \pi(\Omega^{(t)})u(C^*_{g,t}(\Omega^{(t)}))\Big)\Big] - \\ \beta^{t-1}\Big[\sum_{t=2}^{20}\sum_{\Omega^{(t)}} & \pi(\Omega^{(t)}) pen.\Big(f(\theta,\lambda_{max},H_{s,t}(\Omega^{(t)}),b_{max}) - f(\theta,\lambda^*_t(\Omega^{(t)})),H_{s,t}(\Omega^{(t)}),b^*_t(\Omega^{(t)}))\Big)\Big]\Bigg) \end{aligned}$$

where $\pi(\Omega^{(t)})$ is the probability of history $\Omega^{(t)}$.

$V^{(1)}(X_{f,1},H_{s,1},PF_{f,1})$ can be written iteratively in terms of current optimal consumptions $C^*_{f,1}$, $C^*_{s,1}$, optimal time fraction, λ^*_1, optimal books, b^*_1 and expected future (second period) value function $\mathbf{E}V^{(2)}\Big(X^*_{f,2}(\Omega^{(2)}),H^*_{s,2}(\Omega^{(2)}),PF^*_{f,2}(\Omega^{(2)})\Big)$ where $X^*_{f,2}(\Omega^{(2)})$, $PF^*_{f,2}(\Omega^{(2)})$ and $H^*_{s,2}(\Omega^{(2)})$ are the resources-in-hand, contributed-pension-fund deposit and human capital levels for history $\Omega^{(2)}$.

$$\begin{aligned} V^{(1)}(X_{f,1},H_{s,1},PF_{f,1}) = & u(C^*_{f,1}) + \rho u(C^*_{s,1}) - pen.\Big(f(\theta,\lambda_{max},H_{s,1},b_{max}) - f(\theta,\lambda^*_1,H_{s,1},b^*_1)\Big) + \\ & \beta\mathbf{E}V^{(2)}\big(X^*_{f,2}(\Omega^{(2)}),H^*_{s,2}(\Omega^{(2)}),PF^*_{f,2}(\Omega^{(2)})\big) \end{aligned}$$

where

$$\begin{aligned} V^{(2)}(X_{f,2}(\Omega^{(2)}),H_{s,2}(\Omega^{(2)}),PF_{f,2}(\Omega^{(2)})) = & u(C^*_{f,2}(\Omega^{(2)})) + \rho u(C^*_{s,2}(\Omega^{(2)})) - \\ & pen.\Big(f(\theta,\lambda_{max},H_{s,2}(\Omega^{(2)}),b_{max}) - f(\theta,\lambda^*_2(\Omega^{(2)}),H_{s,2}(\Omega^{(2)}),b^*_2(\Omega^{(2)}))\Big) + \\ & \beta\mathbf{E}V^{(3)}\big(X_{f,3}(\Omega^{(3)}\cap\Omega^{(2)}),H_{s,3}(\Omega^{(3)}\cap\Omega^{(2)}),PF_{f,3}(\Omega^{(3)}\cap\Omega^{(2)})\big) \end{aligned}$$

Or,

$$\begin{aligned} V^{(2)}(X_{f,2}(\Omega^{(2)}),H_{s,2}(\Omega^{(2)}),PF_{f,2}(\Omega^{(2)})) = & u(C^*_{f,2}(\Omega^{(2)})) + \rho u(C^*_{s,2}(\Omega^{(2)})) - \\ & pen.\Big(f(\theta,\lambda_{max},H_{s,2}(\Omega^{(2)}),b_{max}) - f(\theta,\lambda^*_2(\Omega^{(2)}),H_{s,2}(\Omega^{(2)}),b^*_2(\Omega^{(2)}))\Big) + \\ & \beta\sum_{\Omega^{(3)}\cap\Omega^{(2)}} \pi(\Omega^{(3)}/\Omega^{(2)})V^{(3)}\big(X_{f,3}(\Omega^{(3)}\cap\Omega^{(2)}),H_{s,3}(\Omega^{(3)}\cap\Omega^{(2)}),PF_{f,3}(\Omega^{(3)}\cap\Omega^{(2)})\big) \end{aligned}$$

Similarly for any period t,

$$V^{(t)}(X_{f,t}(\Omega^{(t)}),H_{s,t}(\Omega^{(t)},PF_{f,t}(\Omega^{(t)})) = u(C^*_{f,t}(\Omega^{(t)})) + \rho u(C^*_{s,t}(\Omega^{(t)})) -$$
$$pen.\Big(f(\theta,\lambda_{max},H_{s,t}(\Omega^{(t)}),b_{max}) - f(\theta,\lambda_t^*(\Omega^{(t)}),H_{s,t}(\Omega^{(t)}),b_t^*(\Omega^{(t)}))\Big)+$$
$$\beta \sum_{\Omega^{(t+1)}\cap\Omega^{(t)}} \pi(\Omega^{(t+1)}/\Omega^{(t)})V^{(t+1)}\big(X_{f,t+1}(\Omega^{(t+1)}\cap\Omega^{(t)}),H_{s,t+1}(\Omega^{(t+1)}\cap\Omega^{(t)}),PF_{f,t+1}(\Omega^{(t+1)}\cap\Omega^{(t)})\big)$$

Since, the wage shocks and stock shocks are individually uncorrelated over time, Ω_t is also uncorrelated over time. Hence, the probability of joint history $\Omega^{(t+1)}$ given Ω^t, $\pi(\Omega^{(t+1)}/\Omega^{(t)}) = \pi(\Omega_{t+1})$ i.e its not dependent on the history $\Omega^{(t)}$ or state Ω_t.
Hence, $\Omega^{(t)}$ is not a state variable and does not influence $V^{(t)}$ beyond the values of $X_{f,t}$, $PF_{f,t}$ and $H_{s,t}$. So, $V^{(t)}(X_{f,t}(\Omega^{(t)}),H_{s,t}(\Omega^{(t)}),PF_{f,t}(\Omega^{(t)})) =$ $V^{(t)}(X_{f,t},H_{s,t},PF_{f,t})$.

Hence, the above equation can be written as,

$$V^{(t)}(X_{f,t},H_{s,t},PF_{f,t}) = u(C^*_{f,t}) + \rho u(C^*_{s,t}) - pen.\Big(f(\theta,\lambda_{max},H_{s,t},b_{max}) - f(\theta,\lambda_t^*,H_{s,t},b_t^*)\Big)+$$
$$\beta \sum_{\Omega_{t+1}} \pi(\Omega_{t+1})V^{(t+1)}\big(X_{f,t+1}(\Omega_{t+1}),H_{s,t+1}(\Omega_{t+1}),PF_{f,t+1}(\Omega_{t+1})\big) \quad (7)$$

3 Benchmark Parameters

3.1 The Three Benchmark Income Levels

The attempt of this study is to solve the consumption, savings, human-capital investments (time and books), and portfolio choice (if applicable) for the entire life-cycle.

For analysis, three education levels of the father have been considered :

1. The Uneducated Dad $\Big(H_{dad} = 1.0\Big)$
2. The Matric Dad $\Big(H_{dad} = 2.0\Big)$
3. The Graduate Dad $\Big(H_{dad} = 5.0\Big)$

Since this study assumes a linear relationship between expected income and human-capital level, the empirically reported ratios of incomes for different education levels have been used to calibrate human-capital levels. As per existing studies, the average income level of an individual with matriculation is roughly twice that of an illiterate person. Hence, the human-capital level of the matric dad is assumed as twice of an uneducated dad. The graduate dad's human-capital level was calculated based on empirically reported average returns (to human-capital) of 11% for an individual with graduation level of education.

It needs to be emphasized that actual income variation in the population is much higher than five times due to a host of factors like permanent shocks, transitory shocks, experience levels, OJT etc. However, ceteris paribus the average income of all individuals with graduation level of education would be roughly five times the average income of all individuals with no-education.

Though this model only considers transitory income shocks, including permanent shocks would not improve the quality of results beyond modelling of the additional impact of shock persistence over time. For the father solving the endogenous human-capital problem of his offspring at $t = 1$, there is an equal chance of a positive permanent shock to the working son twenty years later as a negative permanent shock. In the absence of prior information on the nature of shock, the population averages of incomes for different education levels still remains the crucial variable for human-capital acquisition decision.

Pension-fund savings are ignored in the benchmark scenarios but will be considered later in the next chapter.

3.1.1 The Uneducated Dad

The uneducated dad, with zero years of schooling, is assumed to have the base level of human-capital, $\bar{H} = 1$. Therefore, his income at any given time t is $\bar{G}^{t-1}\bar{H}e_t$ (where e_t is the wage shock). The household is financially excluded and the savings yield negative real returns.
The household gets a 50% subsidy on books and the maximum investment the household can make on books in any period is capped at $b_{max} = 0.25$. The cap on investment is to reflect the scenario of limited access to good schools or other education aids based on income criterion. Pension-fund savings, are not applicable for this income group.

3.1.2 The Matric Dad

The matric dad is assumed to have a human-capital level $H_{matric} = 2$. His income at any given time t is $\bar{G}^{t-1}H_{matric}e_t$. He is assumed to have limited financial access (only bond market access).

This household gets a 50% subsidy on books and the maximum investment the household can make on books in any period is capped at $b_{max} = 0.5$.
Pension-fund savings, as mentioned before, are ignored for now.

3.1.3 The Graduate Dad

The graduate dad is assumed to have a human-capital level $H_{grad} = 5$. His income at any given time t is $\bar{G}^{t-1}H_{grad}e_t$. He is assumed to have full financial access (both bond and stock access).

This household also gets a 50% subsidy on books and the maximum investment the household can make on books in any period is capped at $b_{max} = 1$. Pension-fund savings, as mentioned before, are ignored for now.

The summary of the three benchmark scenarios is presented in Table 1. In all the three scenarios the son's value function is computed with bond rates so as to keep it uniform. Since the focus is on the father's life-cycle decision problem, the attempt is to ensure that differences in son's life-cycle values should arise only out of differences in human-capital levels.

Table 1: The three benchmark scenarios

	DadHumCap	C_{subs}	Fin Access	Book-Subsidy	Book-Max	Pension-Fund
UnEduDad	1.0	0.65	None	50%	0.25	NA
MatricDad	2.0	0.65	Bond	50%	0.5	0
GraduateDad	5.0	0.65	Full	50%	1.0	0

3.2 Consumption and Human Capital Parameters

The utility from consumption has been modelled as a standard CRRA utility,

$$u(C) = C^{1-\xi}/(1-\xi) \tag{8}$$

with the assumption of a moderately risk averse individual with $\xi = 5$.

The Ben-Porath (1967) model was used for the human capital production function, where:

$$\begin{aligned} H_{t+1} = (1-\delta)H_t + \theta(\lambda_t)^{aa}(H_t)^{bb}(b_t)^{cc} \qquad & 6 \le \mathbf{t} \le 20 \\ H_{t+1} = H_t = \bar{H} \qquad & \mathbf{t} < 6 \end{aligned} \tag{9}$$

$\bar{H}$ is the base level of human-capital which is normalized to 1. The minimum adult expected wage corresponds to that of an adult with base level of human-capital. It is equal to 1 at $t = 1$[3]. In monetary terms 1 would be equivalent to Rs 36,000

[3] it grows exogenously at growth rate $\bar{G}$

per year (Rs 100/day).

Since the nutrition effects on human-capital are not modelled in the pre-school stage, it is assumed that the human-capital remains at the base level upto age 6. Also, since the father is solving the life-cycle model at $t = 1$ and assumes that the son will be a separate household once he reaches age of 21, the father only takes into account formal education upto graduation. Human-capital accumulation prospects of the independent son through On-the-Job Training (OJT) is not modelled.

Ability, θ and the other human-capital production coefficients, aa, bb, cc were used from Manuelli & Seshadri (2005) with minor modifications to give an average return of 11% when the child studies full-time with the maximum available books, $b_{max} = 1$. $\theta = 0.16$, the production coefficient values are, $aa = bb = 0.75$, $cc = 0.2$ and the depreciation of human-capital, $\delta = 0.018$.

The father's downward altruism coefficient ρ is assumed to be 1 in the joint phase and 0.7 for the rest of the independent son's life-cycle. As discussed in the previous section, even though the son consumes less than the father, we assume father-son consumption expenditures as equal ($\rho = 1$) in the joint phase since we are not explicitly accounting for health and other expenses of the child.

Several studies in literature have reported high correlations between parent and child's educational attainment[4]. The per-period guilt penalty, pen, which the father feels per-unit of foregone human capital, was calibrated to ensure that on average a father with matriculation level of human-capital ($H_{dad} = 2$) would teach his son upto the matriculation level. After, calibration the minimum penalty value was set to $pen = 1.6$.

Table 2: Benchmark : Consumption and human capital parameters

ξ	θ	aa(=bb)	cc	δ	ρ	subsidy	pen	$\bar{H}$	H_{matric}
5	0.16	0.75	0.2	1.8%	1/0.7	50%	1.6	1	2

3.3 Financial Parameters

A bond rate of $r_0 = 4\%$[5] is assumed and expected stock returns are assumed at $\mathbf{E}(r_s) = 9\%$[6]. The returns for the financially excluded uneducated dad household is assumed as $r_{nofin} = -5\%$. The variance of innovation to risky asset (stock shock) is taken as $var(\gamma_t) = 0.0246$ and the variance of transitory wage shocks e_t, has been calibrated at 0.085. (The values of stock and wage shock variance are close to the values in Cocco et al. (2005)) The correlation between income shocks and stock returns, has been assumed as $corr = 1$.
Discount rate is set as $\beta = 0.98$. Contributed-pension-fund savings rate,τ, whenever applicable, is 5% and the father leaves 15% of his last period's wealth as planned bequest to his son. Contributed-pension-fund deposits, if applicable, earn bond interest rates and are risk-free.
The exogenous growth rate of income is assumed at $\bar{G} = 1\%$.

Table 3: Benchmark : Financial parameters

r_0	$\mathbf{E}r_s$	r_{nofin}	var(γ_t)	var(e_t)	corr	β	τ	Bequest	$\bar{G}$
4%	9%	−5%	0.0246	0.085	1	0.98	5%	15%	1%

3.4 Life-Cycle and Child-Labour Parameters

The model starts at $t = 1$. The son starts going to school or work at $t = 6$. The wage rate of the child between $6 \leq t \leq 10$ is 33% of the minimum adult wage rate. The wage rate of the son between $11 \leq t \leq 15$ is 50% of the minimum adult wage

[4] Ermisch & Francesconi (2001), Woessmann (2004), Black et al. (2003)
[5] $R_0 = 1.04$
[6] $\mathbf{E}R_s = 1.09$

rate. From $16 \leq t \leq 20$ the wage rate is 100% of the minimum adult wage rate if the human-capital level of the son is less than H_{matric} or else its matriculation wage rate (which is 200% of the minimum adult wage rate). Child-labour wages are assumed to be non-stochastic.

The son becomes an adult at the age of 21 ($t = 21$) and earns a wage rate reflecting his true human-capital level and starts his own family of two members. Adult (age ≥ 21) wages are subject to wage shocks. The father retires at the age of 61 ($t = 41$) and from that year onwards receives the annuity value of his accumulated pension-fund deposit till he dies. The father's last period is at age 70 ($t = 50$) and the son receives the bequest in the next period.

The son retires at $t = 61$ and dies at $t = 70$ which is also the last period of the model.

4 Simulation Results : Benchmark Scenarios

This chapter presents the simulation results for the three benchmark scenarios explained in Chapter-4. In order to generate average consumption and human-capital investment profiles repeated simulations trials were done by drawing numerous realizations of the income and stock return shocks. The life-cycle profile values reported in this chapter are the cross-section means of 2000 trials.

5 The Uneducated Dad ($H_{dad} = 1$)

The per-capita consumption levels and educational investments of the household are presented in Table 4.

The simulation output (Figure 3) shows that the per-capita consumption in the first 5 years when the son is at pre-school stage, $C_{joint(1-5)}$, is very low. The moment the son can be sent to work the average per-capita consumption rises. Hence, the family finds the child-labour option optimal as it helps raise the per-capita consumption levels to a minimum level. After the son starts work the father starts to save small amounts every period. Since, the father does not have access to financial resources his savings yield negative returns. The independent dad's average per-capita consumption level, $C_{dad(21-50)}$[7], owing to not having to bear any-more the son's consumption expenses, is somewhat higher and he leaves a small bequest to his son.
The educational inputs (education time and books) and the human-capital levels of the son have been shown in Figures 4

Table 4: Uneducated Dad : Benchmark values

	$C_{joint(1-5)}$	$C_{joint(1-20)}$	$C_{dad(21-50)}$	$C_{son(21-70)}$	Beq	Edu(yrs)	Books(Tot)	H_{son}
UnEduDad	0.5	0.653	0.717	1.054	0.12	0	0	1.0

Per-capita consumption values mentioned. $C_{subs} = 0.65$. 1.0 is equivalent to Rs 36,000.
Books(Tot) refers to total investment in books over the entire education phase.

and 5. The son is never sent to school and the investment in books is zero. As a result, the son's human capital stays at the base level $\bar{H} = 1$.

Figure 6 shows the father's income profile, per-capita consumption levels, child-labour earnings, books expenses and the asset levels of the household for the first 20 joint periods. Figure 7 extends Figure 6 to seventy periods of the life-cycle with additional plots of independent dad's income, consumption and the independent son's consumption.

It is clear from Figures 6 and 7 that even though the net income[8] of the household fluctuates in the joint-periods with changing child-labour wage rates, the per-capita consumption remains smooth and monotonic, except in the first five periods when the family is binding constrained and net consumption matches net income. The father starts building assets from the period the son can be sent to work. The steepness of the asset curve changes with changes in child-labour wage rates. As the wage rate of child-labour increases the family savings increase commensurately, keeping the consumption smooth. The father's asset levels continue to monotonically increase through mid-life and reach maximum the year before

[7] $21 \leq t \leq 50$
[8] dad's income plus childlabour income net of book expenses

his retirement and then falls steeply. The independent father's consumption maintains the smooth trend, even through the post-retirement periods when his income has fallen to zero.

The independent son's consumption is piecewise smooth from $t = 21$ to 40 and $t = 41$ to 70. The two jumps is son's per-capita consumption from $t = 20-21$ and $t = 40-41$ are a consequence of borrowing constraints. Similar jumps are also seen in the other two scenarios of matric and graduate dad. A detailed analysis of the jumps in son's consumption is discussed in the graduate dad section.

The life-cycle optimization model confirms that child-labour is one of the ways of very poor households to increase consumption levels to the bare minimum subsistence level. As a result, the child's future gets traded-off and the son, like father, starts his adulthood with no education. Without external intervention, this would be a self-perpetuating inter-generational poverty trap.

The incidence of child-labour is very high in the Third World Countries. According to the Bureau of Statistics, International Labour Organization, in 2012 around 168 million children were engaged in child-labour. Half of them were engaged in hazardous work. Tables 5 and 6 summarize the number of children in employment and their distribution across the world. From a policy perspective, without exploring the possibilities of financial inclusion or cash support, banning child-labour might lead to even worse outcomes like hunger and starvation. The next chapter of this thesis explores in greater detail the effects of different financial and human-capital parameters on child labour, consumption and education.

Table 5: Children in employment, child-labour and hazardous work in the world, 5-17 years age group, 2000-2012

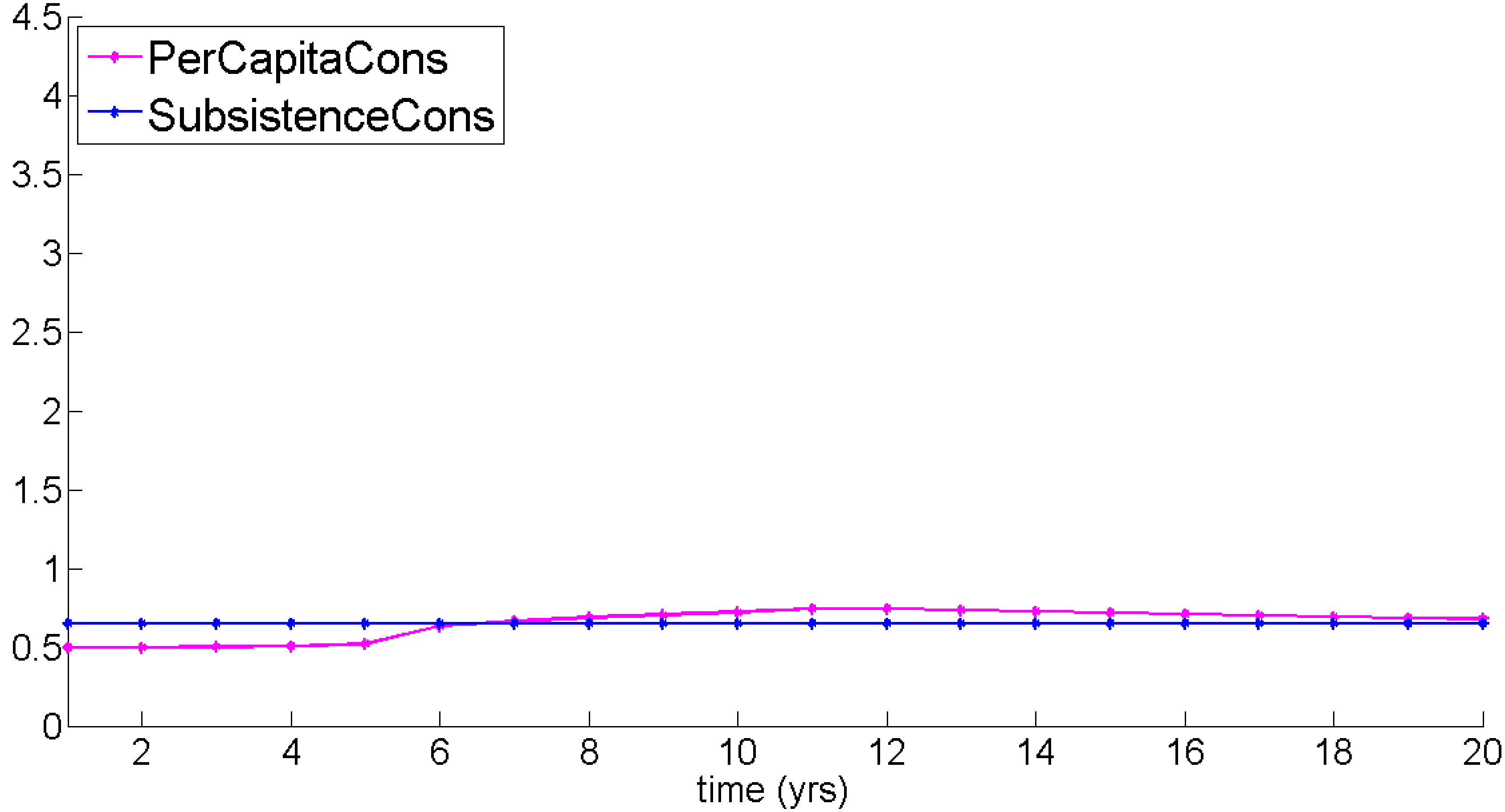
Year	Children in employment	Child labour	Hazardous work
2000	351	245	170
2004	322	222	128
2008	305	215	115
2012	264	167	85

Source: ILO (2012). All figures are in millions.

Table 6: Children in employment, child-labour and hazardous work, 5-17 years age group, 2012

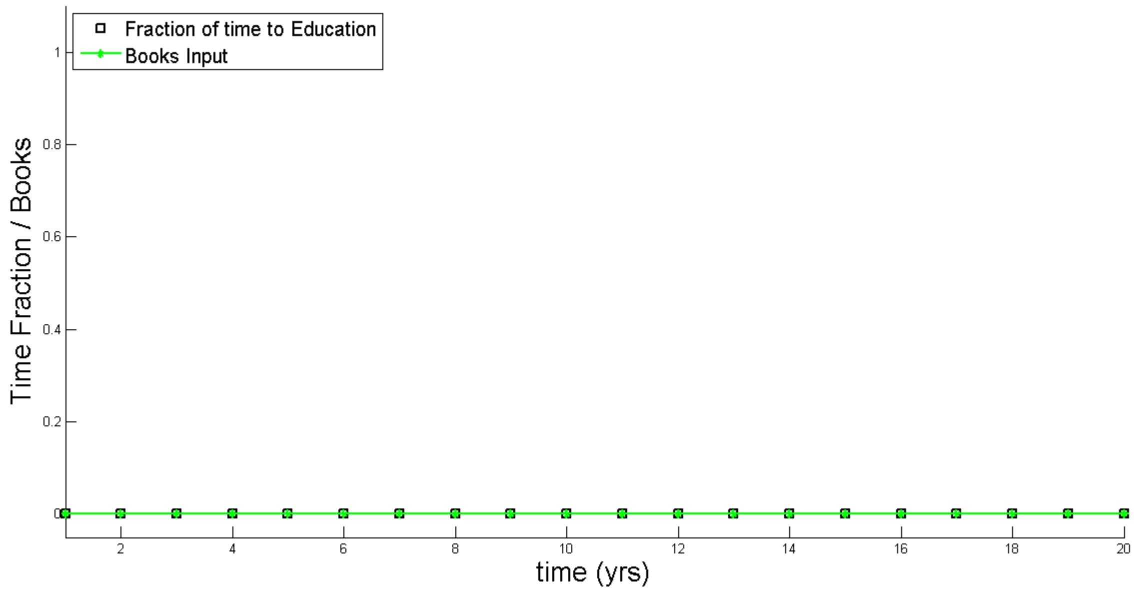
Region	Children in employment	Child labour	Hazardous work
Asia and Pacific	129	77	33
Latin America and Caribbean	12	222	9
Sub Saharan Africa	305	59	28
Middle East and North Africa	13	9	5

Source: ILO (2012). All figures are in millions.



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 3: Uneducated Dad: Per-capita consumption and asset levels in the first twenty joint-periods



Y-axis value for time is a fraction.
Y-axis values for books are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 4: Uneducated Dad: Investments in education time and books

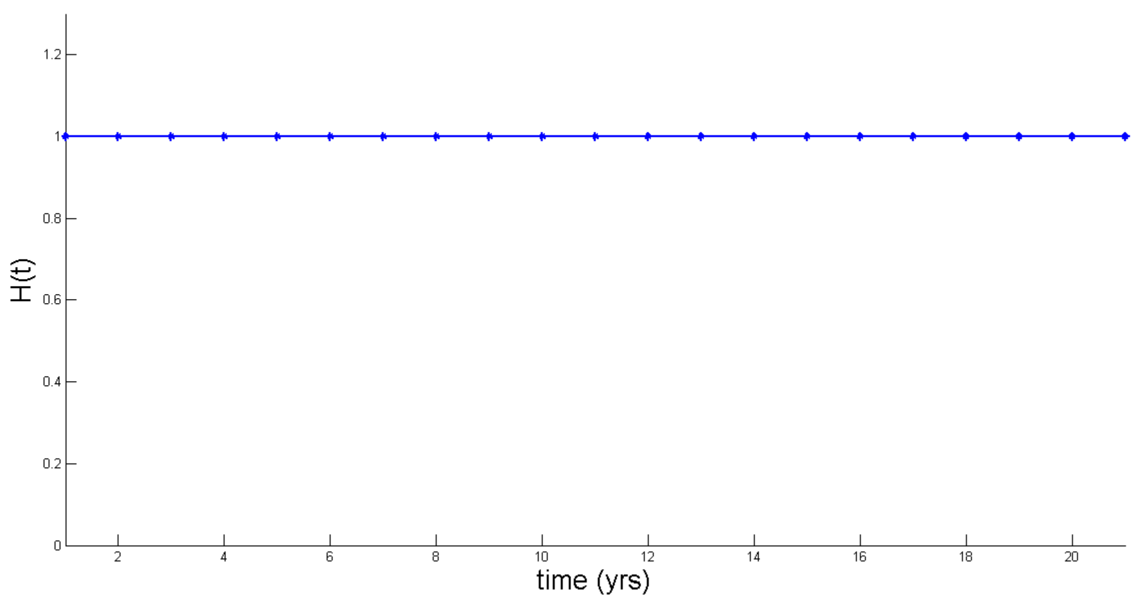
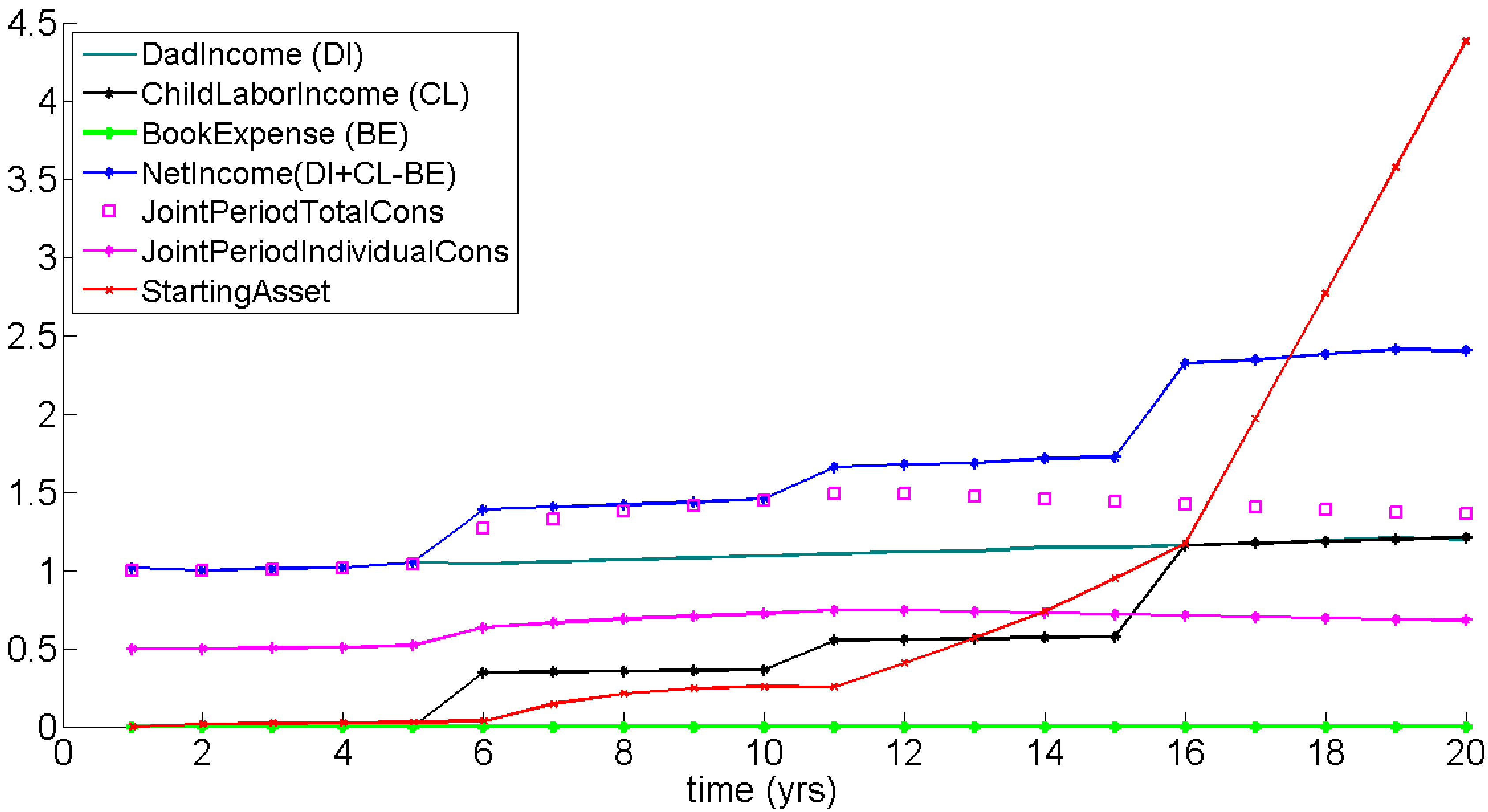
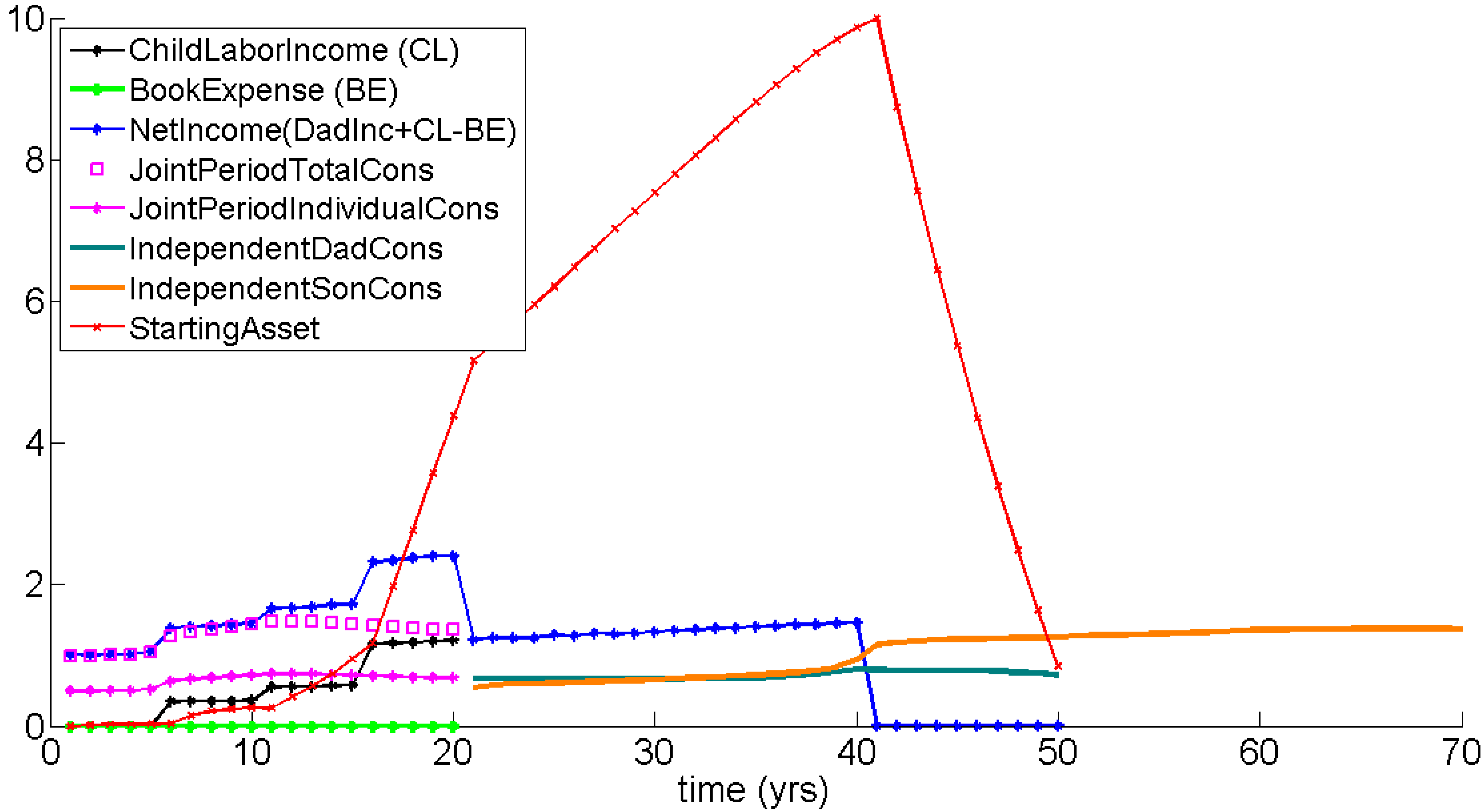


Fig. 5: Uneducated Dad: Human-capital levels of the son



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 6: Uneducated Dad: Life-cycle profiles for the first twenty joint-periods



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 7: Uneducated Dad: Two-generation life-cycle profiles for seventy periods

6 The Matric Dad ($H_{dad} = 2$)

The per-capita consumption levels and educational investments of the matric dad's household are presented in Table 7.

Figure 8 shows that the per-capita consumption is sloping upwards for all 20 periods. The average per-capita consumption over the 20 joint-periods, $C_{joint(1-20)}$, is 1.83 times the per-capita consumption of the Uneducated Dad's household. The independent dad's average consumption level, $C_{dad(21-50)}$[9], owing to lower expenses, is much higher and he leaves a positive bequest.

Figures 9 and 10 show the educational inputs (education time and books) and the human-capital levels of the son.

Table 7: Matric Dad : Benchmark values

	$C_{joint(1-20)}$	$C_{dad(21-50)}$	$C_{son(21-70)}$	Beq	Edu(yrs)	Books(Tot)	H_{son}
MatricDad	1.201	2.117	2.157	0.388	7.525	3.281	2.046

Per-capita consumption values mentioned. $C_{subs} = 0.65$. 1.0 is equivalent to Rs 36,000.
Books(Tot) refers to total investment in books over the entire education phase.

The son gets partial education during all periods. In the initial periods when the assets are low, the fraction of time devoted to education is lower but it increases gradually till upto $t = 10$. From $t = 11$, the opportunity cost of education increases owing to a higher child-labour wage rates, leading to a reconfiguration of investment in time and books according to equation $\left(b^* = (\bar{G}^{t-1} H_{relv}/s)(cc/aa)\lambda_t^*\right)$. A similar trend gets repeated from $t = 16$ when again, the child-wage rates change. Only in the last period, $t = 20$, just before entering the adult labour market, the son studies full time ($\lambda = 1$) and the investment in books is also the maximum available at $b_{max} = 0.5$. The over-investment in education in the last period occurs because, as per the assumptions of the model, the actual returns to education come only at adulthood (age of 21) when the wage reflects the true human capital level. Hence, due to discounting and depreciation, for the same level of target human-capital, a configuration of higher investments in later periods would be chosen. This in reality may be viewed as extra-tuitions or crash courses that are typically taken just before a critical exam like matriculation, higher-secondary or graduation. In all, the total investment of education time is 7.5 years and the total investment in books is 3.281. The son enters adulthood ($t = 21$) with a human-capital of $H_{son} = 2.046$.

Figure 11 shows the father's income profile, per-capita consumption level, child-labour earnings, books expenses and the asset levels of the household for the first 20 joint periods. Figure 12 extends Figure 11 to seventy periods of the life-cycle with additional plots of independent dad's income, consumption and the independent son's consumption.

Starting from $t = 1$, the asset levels first increase in the initial years[10] as consumption levels are lower (Figure 11). The assets then start to decrease[11] as educational investments increase owing to more time fraction spent studying and higher book expenses. The assets follow a similar trend starting from $t = 16$, when the child-wage rate enters the next slab. Figure 12 shows that the independent father starts with zero assets and his assets form a humped shape from $t = 21$ to 50 reaching maximum at $t = 40$[12].

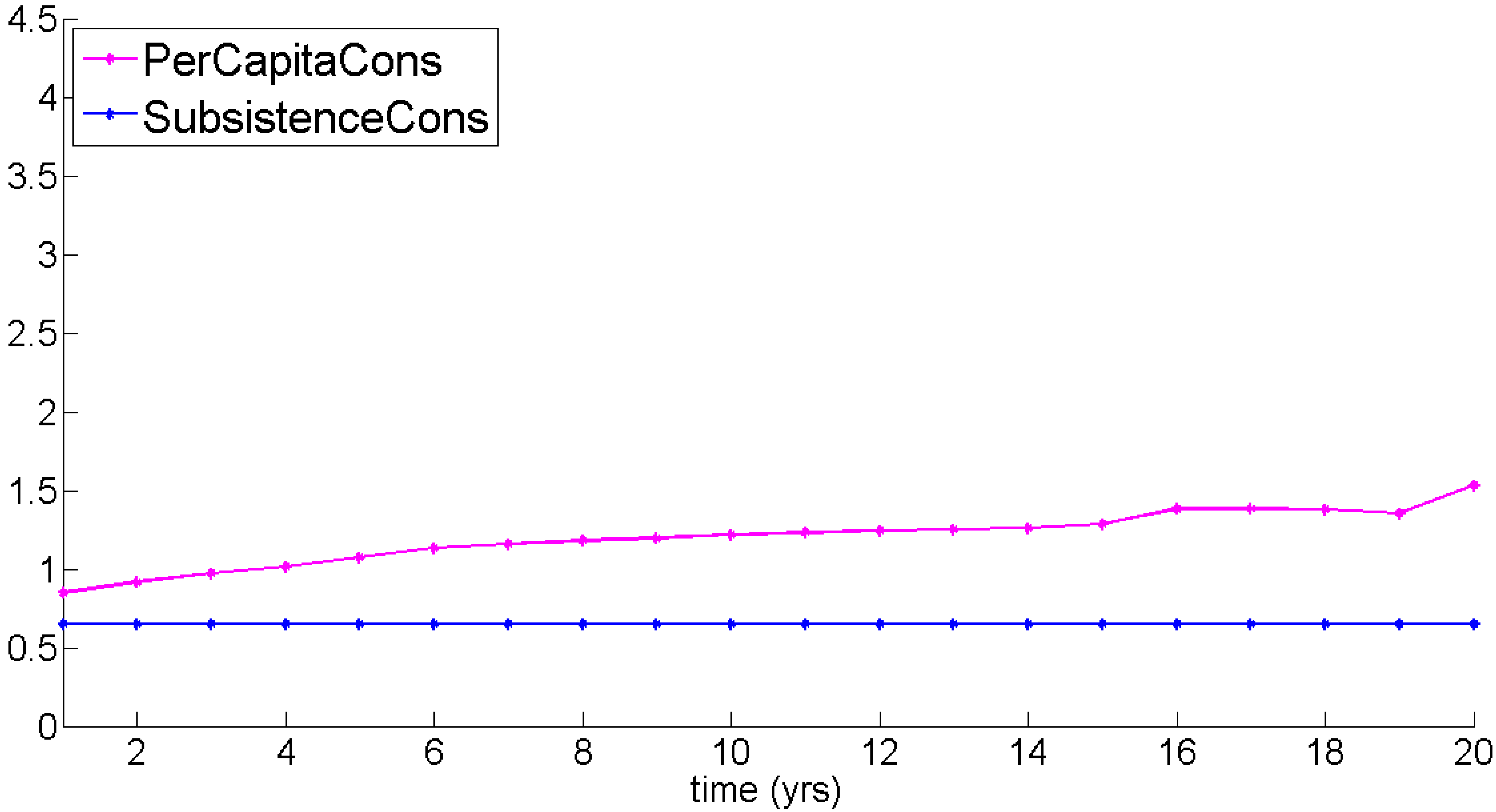
It is clear from Figures 11 and 12 that even though the net income[13] of the household fluctuates with changing child-labour wage rates, the per-capita consumption levels remain smooth and monotonic. Per-capita consumption of the father and son does not fluctuate as much as net income during the first twenty joint periods. As discussed in the previous paragraph, the father does not carry forward any assets into the independent phase.

A jump from $t = 20$ to $t = 21$ is seen in the per-capita consumptions of both the father and the son. A similar jump can also be seen in the son's consumption from $t = 40$ to $t = 41$. A detailed analysis of the jumps in consumption is presented in the graduate dad section.

[9] $21 \le t \le 50$
[10] reaching maximum at $t = 8$
[11] reaching minimum at $t = 16$
[12] last period before retirement
[13] dad's income plus childlabour income net of book expenses

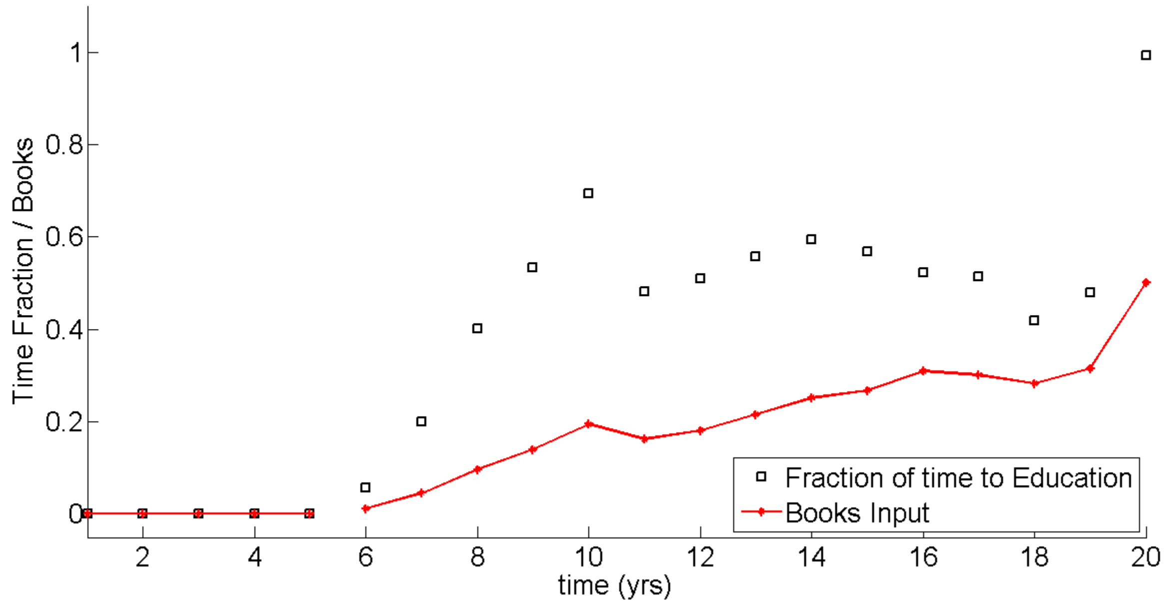
The benchmark scenario reveals the existence of partial education for households with incomes slightly higher than subsistence. The link between household income and education is well documented in empirical literature. A recent paper by Pal (2013) analysing income-related inequality on educational performance using IHDS data set, finds evidence of economic status of the household being an important determinant of educational performance. Their estimate of the contribution of wealth inequality to inequalities in test scores is close to 20%. Similarly, Filmer & Pritchett (2001) attempt to estimate the relationship between wealth and school enrolment using household asset variables on Indian data. Their results show large differences in school enrolment by wealth. Hill & Duncan (1987), using PSID data, also find significant effects of parental family income on completed schooling and wage rates of children.

The next chapter explores the effects of various parameters like financial widening, contributed-pension-fund savings, asset returns, education subsidies and child ability on educational investments. This income category is the most interesting to explore in terms of policy implications as small changes in parameters can have large effects on life-cycle outcomes. The quantification of elasticities of educational investments to parameter changes would reveal the critical variables for policy attention.



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 8: Matric Dad: Per-capita consumption and asset levels in the twenty joint-periods



Y-axis value for time is a fraction.
Y-axis values for books are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 9: Matric Dad: Investments in education time and books

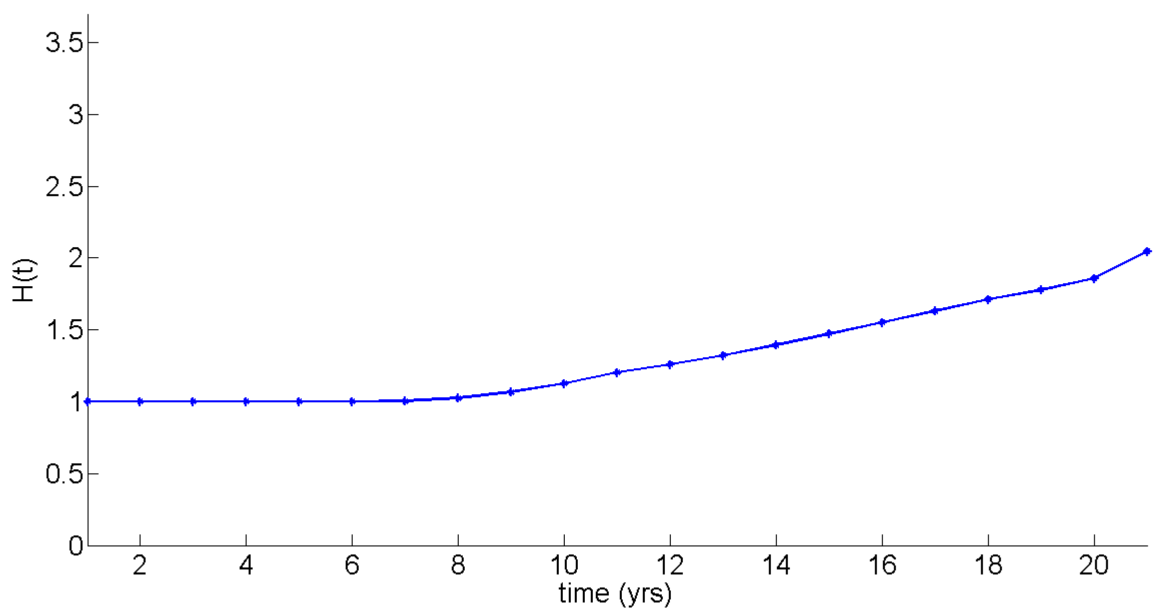
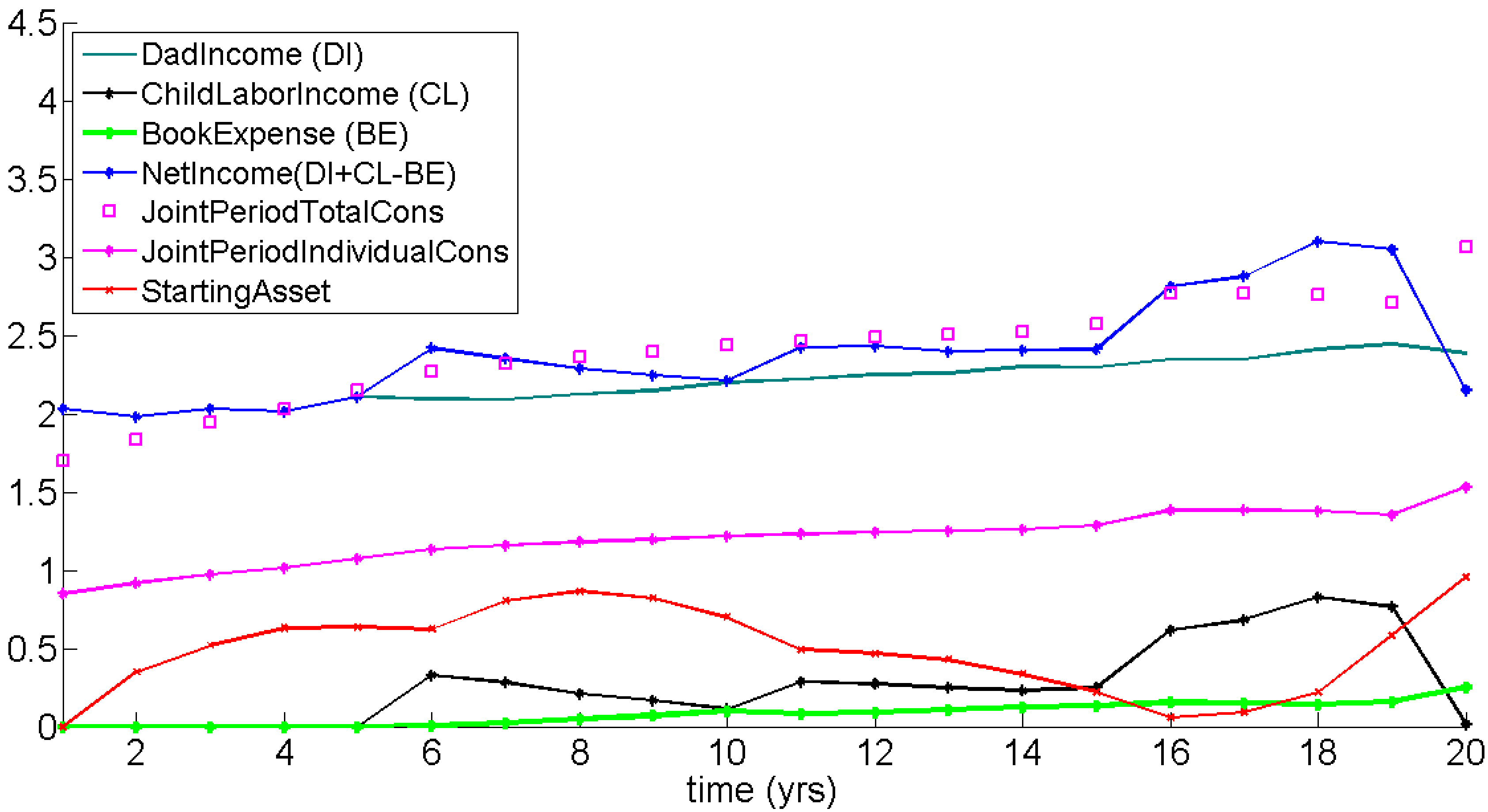
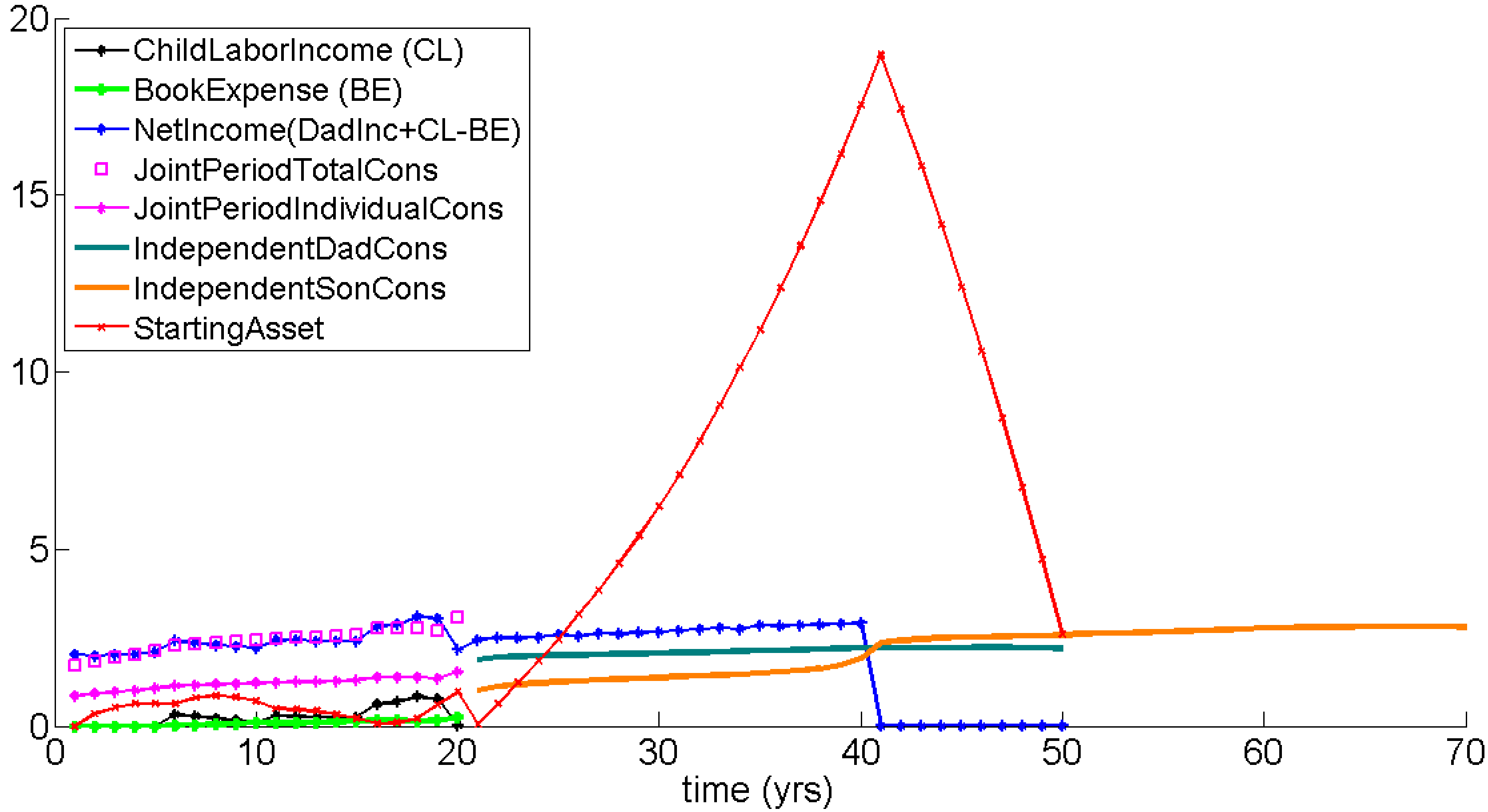


Fig. 10: Matric Dad: Human-capital levels of the son



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 11: Matric Dad: Life-cycle profiles in the first twenty joint periods



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 12: Matric Dad: Two-generation life-cycle profiles for seventy periods

7 The Graduate Dad ($H_{dad} = 5$)

Table 8 presents the per-capita consumption levels and educational investments of the household.

The per-capita consumption is sloping upwards for all 20 periods (Figure 13). The average per-capita consumption over the 20 joint-periods, $C_{joint(1-20)}$, is 2.23 times the per-capita consumption of the Matric-Dad's household. The independent dad's average consumption levels, $C_{dad(21-50)}$[14], owing to discontinuation of son's consumption and educational expenses, are much higher at 6.12 and he leaves a healthy bequest equal to 1.35.

Table 8: Graduate Dad : Benchmark values

	$C_{joint(1-20)}$	$C_{dad(21-50)}$	$C_{son(21-70)}$	Beq	Edu(yrs)	Books(Tot)	H_{son}
GradDad	2.69	6.12	5.3	1.35	15	15	5.011

Per-capita consumption values mentioned. $C_{subs} = 0.65$. 1.0 is equivalent to Rs 36,000.
Books(Tot) refers to total investment in books over the entire education phase.

Figures 14 and 15 show the educational inputs (education time and books) and the human-capital levels of the son. The son of a graduate dad gets full education in all periods. The investment on books is also at the maximum permissible limit of $b_{max} = 1$ in every period. Only higher ability or higher access to educational inputs (higher b_{max}) can increase the human-capital level of the son any further. The son enters adulthood, ($t = 21$) with a human-capital of $H_{son} = 5.011$.

Figure 17 shows the father's income profile, per-capita consumption level, books expenses and the asset levels of the household for the first 20 joint periods. Figure 18 extends Figure 17 to seventy periods of the life-cycle with additional plots of independent dad's income, consumption and the independent son's consumption.

Figures 17 and 18 collectively reveal a two-humped asset profile of the father. Starting from $t = 1$, the assets grow very fast in the first five years and then increase gradually once the son starts going to school. The asset levels peak at $t = 13$ after which they start to decrease to maintain the upward sloping per-capita consumption. Interestingly, negligible assets are carried forward by the father into the next stage resulting in a humped asset profile from $t = 1$ to 21. The savings are built in the first half of the joint periods to maintain the upward sloping consumption profile in the second half. Figure 18 shows that the independent father starts with zero assets. For the father, savings do not seem to be a means of transferring resources to the future periods ($t = 21$ onwards) after the son separates. A second hump in father's assets can be seen from $t = 21$ to 50, reaching maximum at $t = 40$ (period before retirement).

It is evident from Figures 17 and 18 that per-capita consumption levels remain smooth and monotonic through the joint-periods.

Jumps in consumption
A jump from $t = 20$ to 21 is seen in the per-capita consumption of both the father and the son. The jump in the father's consumption stems from the fact that after the son separates, the father's expenses are lower due to discontinuation of the son's consumption and educational expenses and hence, the father readjusts his consumption upwards. The jump would not have arisen in an environment without borrowing constraints as then the father would have borrowed from his independent phase income to increase the per-capita consumption level in the joint-phase. Figure 16 plots for the 50 periods of father's life, the average fraction of times when the father is borrowing constrained in any given period (zero savings). It can be seen from the figure that at the last period before separation ($t = 20$) the father is borrowing constrained in more than 60% of the simulation trials. The household consumes all its resources and the father starts his independent phase with zero assets. The independent father's consumption, after the jump, maintains a smooth trend even through the post-retirement periods when his income has fallen to zero. A similar jump was also witnessed in the Matric dad's consumption in the previous section.

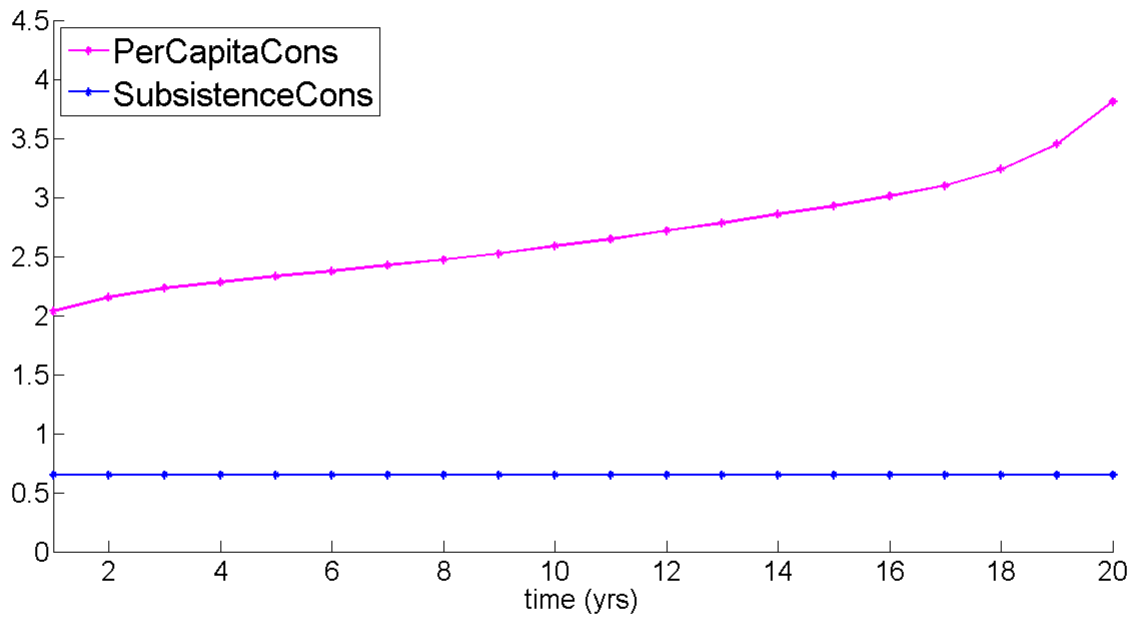
Similarly the independent son's consumption is also piecewise smooth from $t = 21$ to 40 and $t = 41$ to 70. The jump in the son's consumption from $t = 20$ to 21 stems from the model assumption that the son starts his independent life

[14] $21 \leq t \leq 50$

with zero assets. Since, there is no wealth transfer from the father to the independent son,[15] and the son's consumption is adjusted in accordance with his permanent income.

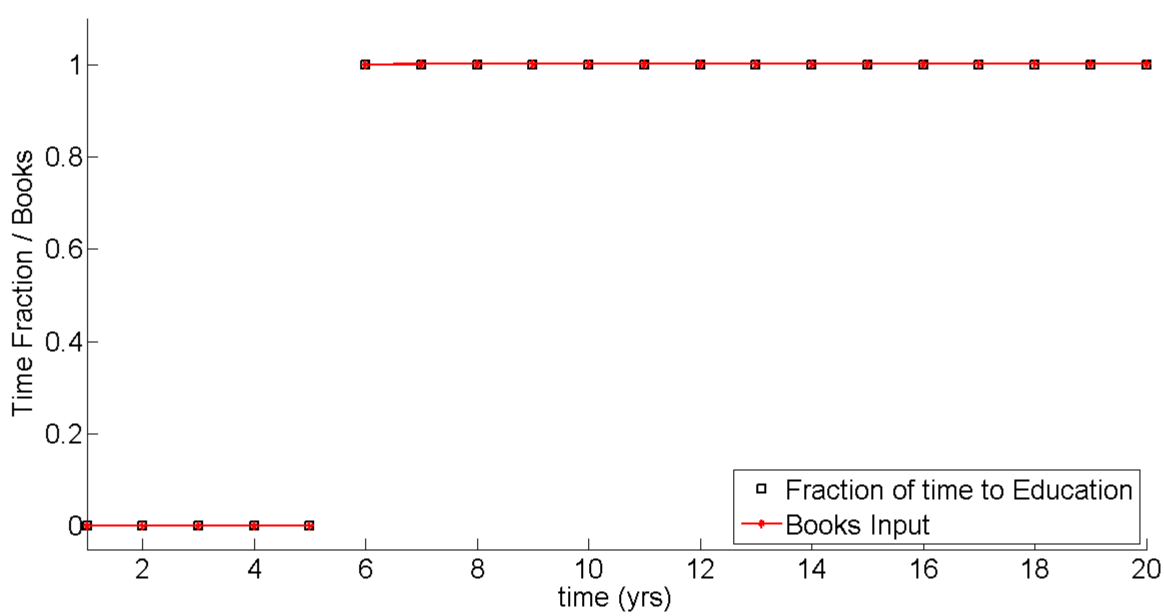
The simulation results confirm that educational investment is not a problem for the children of the highest income category. They get complete education with maximum available inputs. For this income category, consumption, savings and portfolio choice are more crucial. The next section explores the portfolio choice of the father for different correlations between wage and stock shocks. The next chapter explores in further detail the consumption and portfolio choice for changes in financial parameters.

[15] the son receives the bequest much later



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 13: Graduate Dad: Per-capita consumption and asset levels in the twenty joint-periods



Y-axis value for time is a fraction.
Y-axis values for books are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 14: Graduate Dad: Investments in education time and books

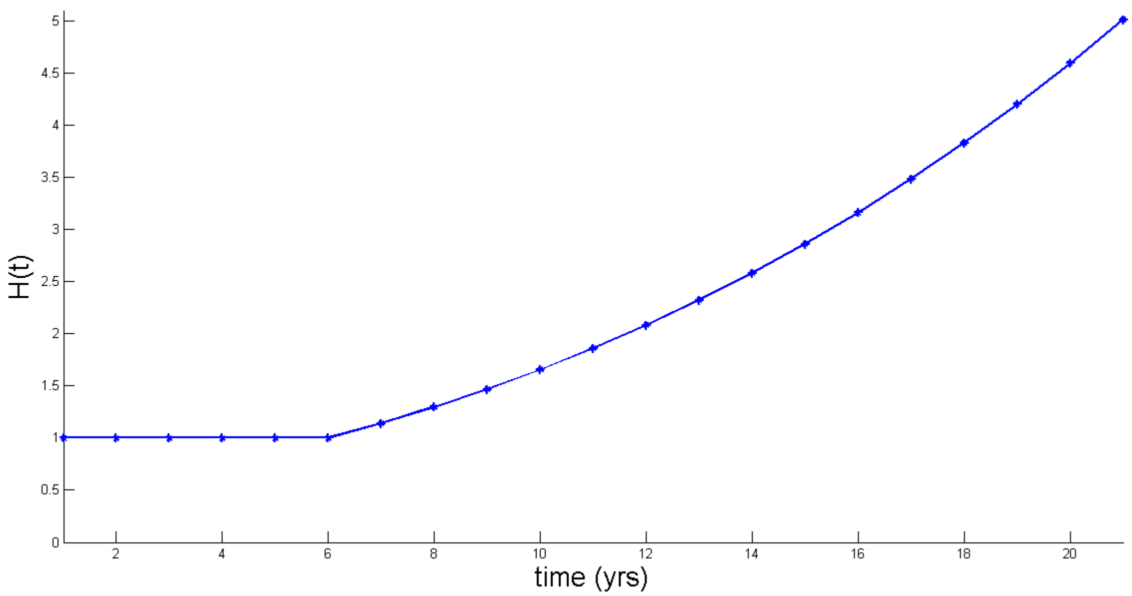


Fig. 15: Graduate Dad: Human-capital levels of the son

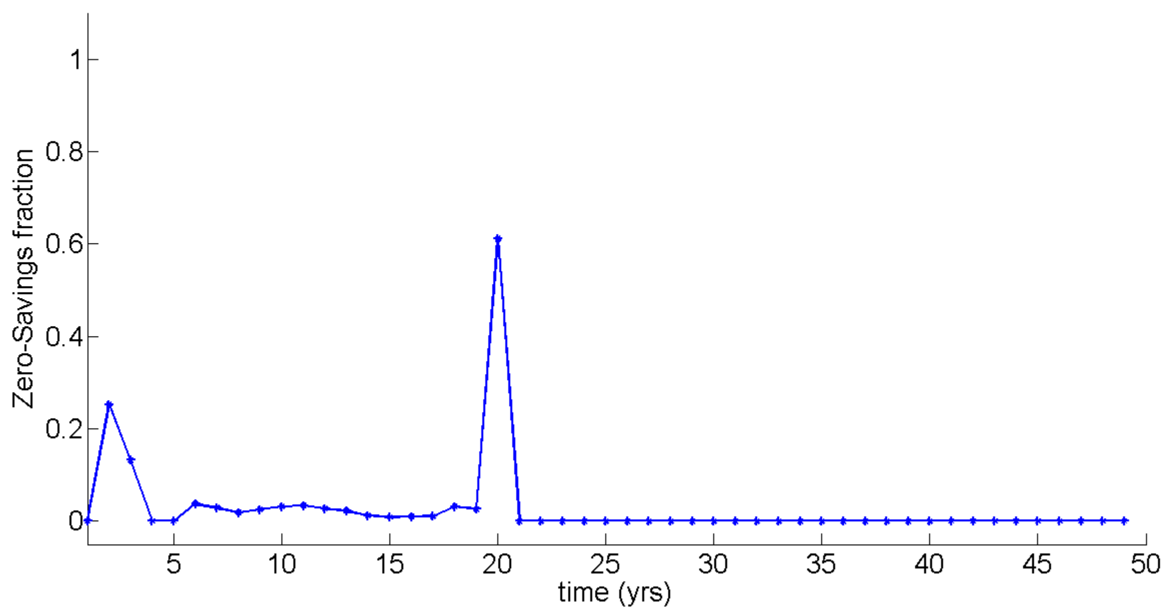
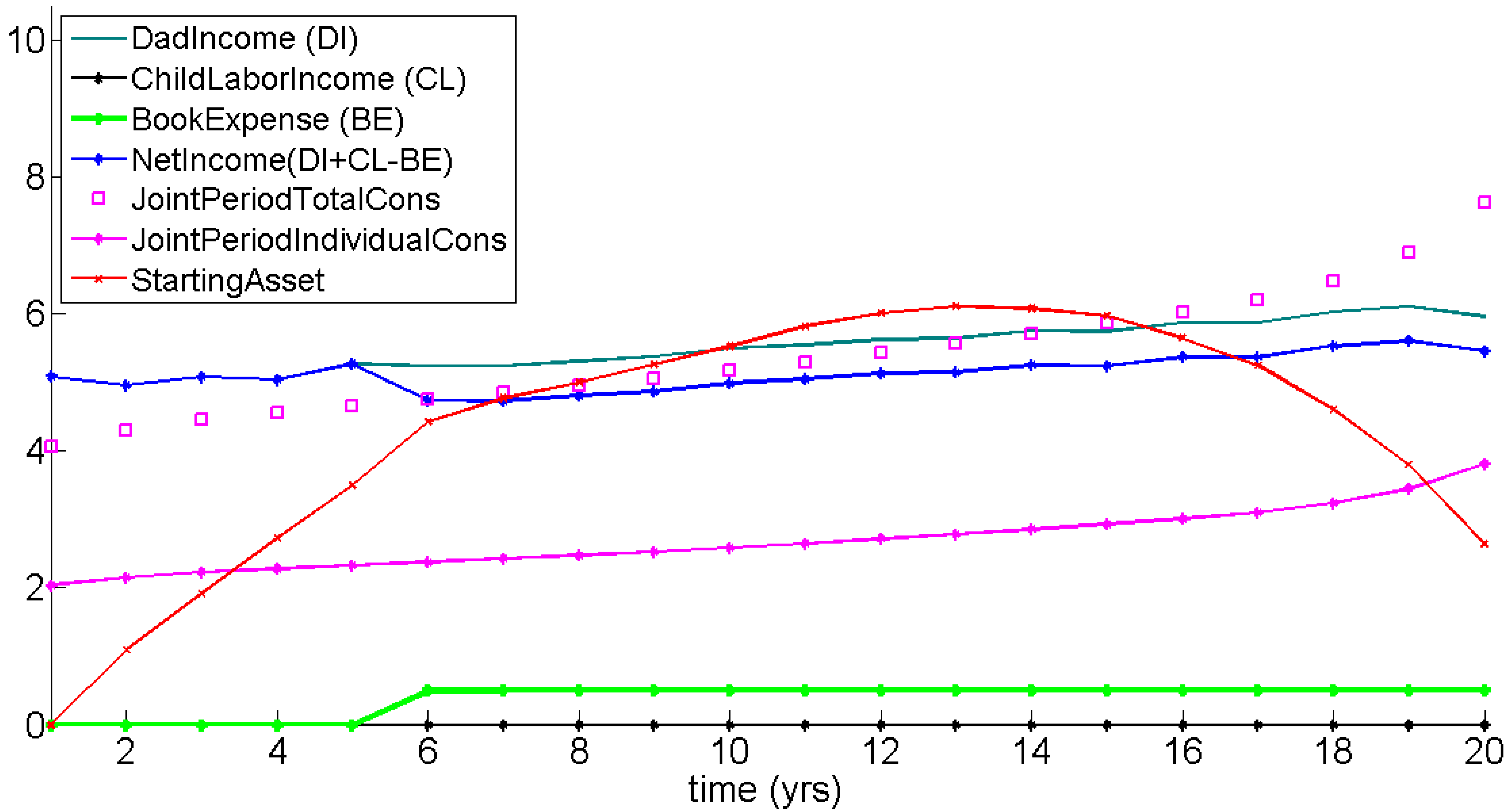
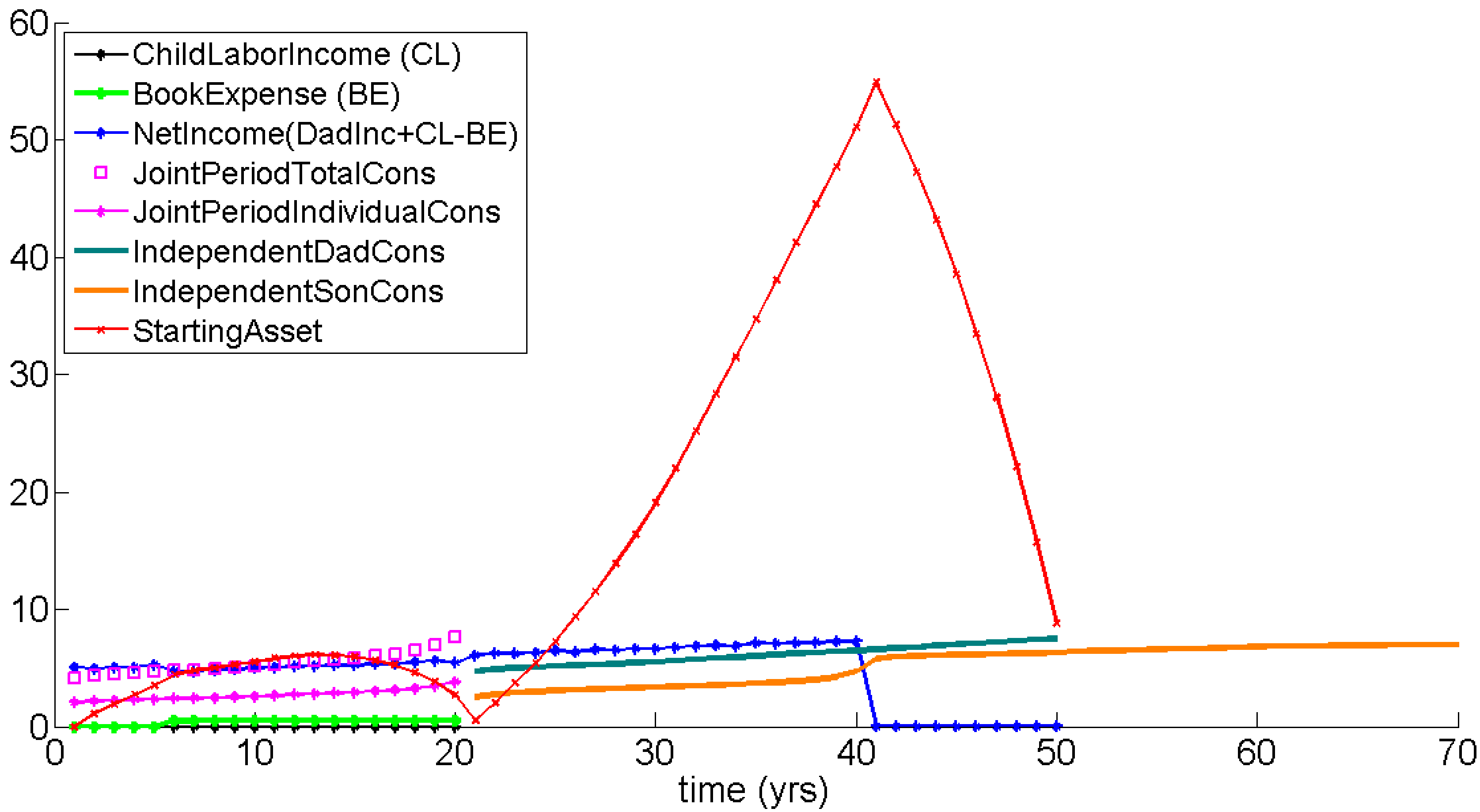


Fig. 16: Graduate Dad: Binding borrwoing constraint: zero savings periods



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 17: Graduate Dad: Life-cycle profiles in the first twenty joint periods



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 18: Graduate Dad: Two-generation life-cycle profiles for seventy periods

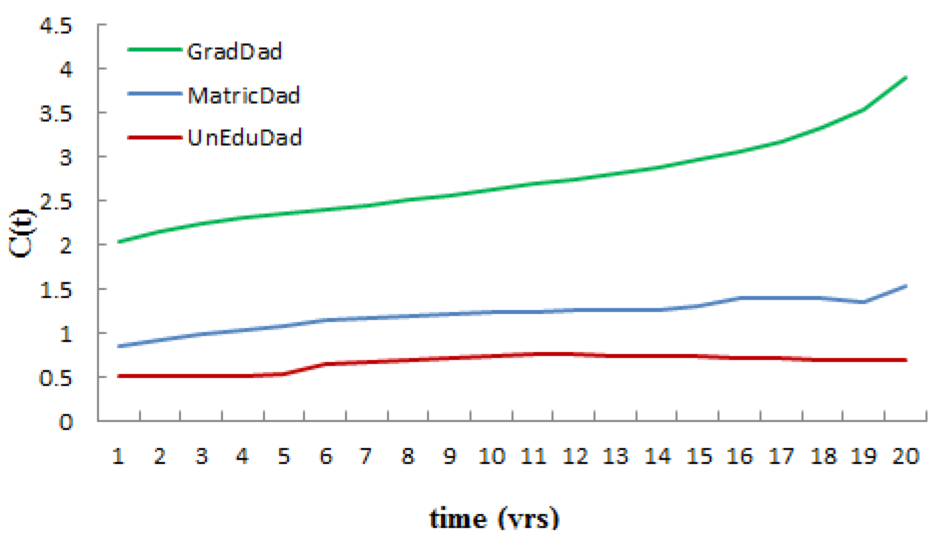
7.1 Comparison of the Three Income Categories

This subsection summarizes the life-cycle consumption, asset levels and human-capital levels of the three income categories discussed in the previous sections. Table 9 presents the consumption values, educational investments and human-capital levels for the three-households while Figures 19, 20 and 21 plot the consumption levels, asset levels and the human-capital levels respectively.

Table 9: Comparison of the consumption levels and educational investments for the three income categories

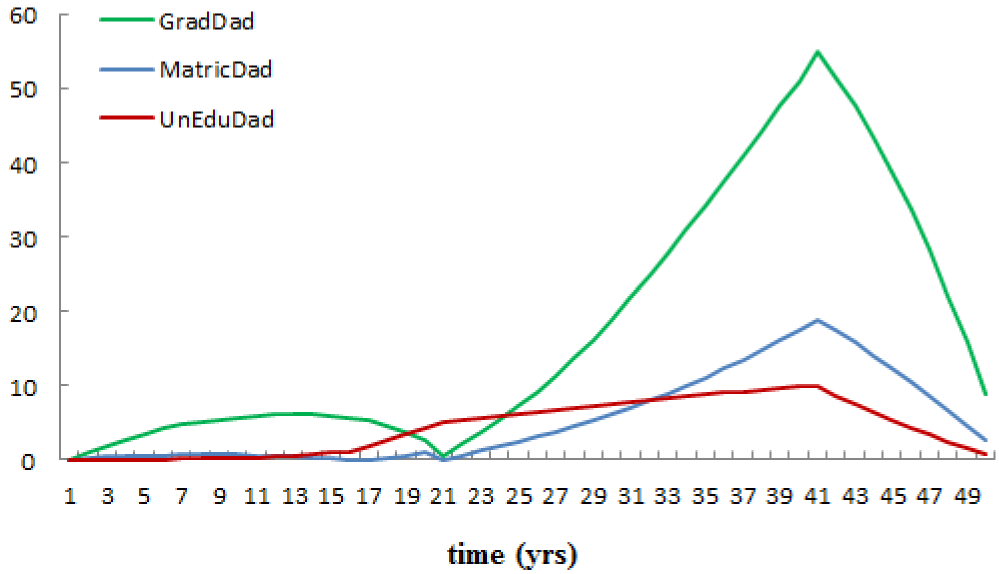
	$C_{joint(1-20)}$	$C_{dad(21-50)}$	$C_{son(21-70)}$	Beq	Edu(yrs)	Books(Tot)	H_{son}
UnEduDad	0.653	0.717	1.054	0.12	0	0	1.0
MatricDad	1.201	2.117	2.157	0.388	7.525	3.281	2.046
GradDad	2.69	6.12	5.3	1.35	15	15	5.01

All consumptions are per-capita. 1.0 is around Rs 36,000
Books(Tot) refers to total investment in books over the entire education phase.



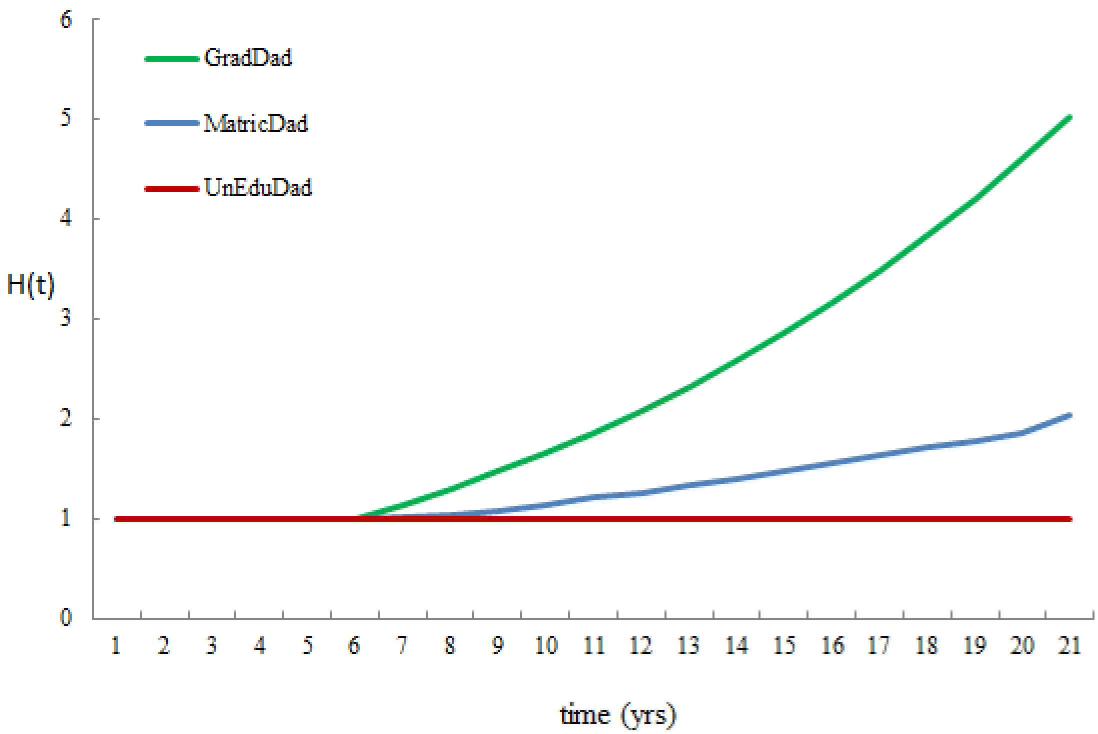
Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 19: Comparison of joint-phase consumption levels of three households



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 20: Comparison of the asset levels of three households



Y-axis values are multiples of minimum adult wage. 1.0 is equivalent to Rs 36,000.

Fig. 21: Comparison of the human-capital levels of three households

References

Becker. (1964). Human capital. *Columbia University Press for the National Bureau of Economic Research. New York: Proyecto Culturas Juveniles*.

Becker, G., & Tomes, N. (1994). Human capital and the rise and fall of families. In *Human capital: A theoretical and empirical analysis with special reference to education (3rd edition)* (pp. 257–298). The University of Chicago Press.

Becker, G. S. (1960). Underinvestment in college education? *The American Economic Review*, *50*(2), pp. 346-354. Retrieved from `http://www.jstor.org/stable/1815038`

Belley, P., & Lochner, L. (2007). *The changing role of family income and ability in determining educational achievement* (Tech. Rep.). National Bureau of Economic Research.

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, *70*(2), 481–517.

Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *The Journal of Political Economy*, *75*(4), 352–365.

Black, S. E., Devereux, P. J., & Salvanes, K. G. (2003). *Why the apple doesn't fall far: Understanding intergenerational transmission of human capital* (Tech. Rep.). National Bureau of Economic Research.

Bohacek, R., & Kapicka, M. (2008). Optimal human capital policies. *Journal of Monetary Economics*, *55*(1), 1–16.

Carneiro, P., & Heckman, J. (2002). The evidence on credit constraints in post-secondary schooling*. *The Economic Journal*, *112*(482), 705–734.

Cocco, J., Gomes, F., & Maenhout, P. (2005). Consumption and portfolio choice over the life cycle. *Review of financial Studies*, *18*(2), 491–533.

Cunha, F., & Heckman, J. (2008). Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation. *Journal of Human Resources*, *43*(4), 738–782.

Cunha, F., Heckman, J., & Schennach, S. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, *78*(3), 883–931.

Del Boca, D., Flinn, C., & Wiswall, M. (2010). Household choices and child development.

Ellwood, D., Kane, T., et al. (2000). Who is getting a college education? family background and the growing gaps in enrollment. *Securing the future: Investing in children from birth to college*, 283–324.

Ermisch, J., & Francesconi, M. (2001). Family matters: Impacts of family background on educational attainments. *Economica*, *68*(270), 137–156.

Filmer, D., & Pritchett, L. H. (2001). Estimating wealth effects without expenditure dataor tears: An application to educational enrollments in states of india*. *Demography*, *38*(1), 115–132.

Friedman, M. (2002). *Capitalism and freedom*. University of Chicago press.

Hill, M. S., & Duncan, G. J. (1987). Parental family income and the socioeconomic attainment of children. *Social Science Research*, *16*(1), 39–73.

Johnson, M. (2010). Borrowing constraints, college enrollment, and delayed entry. *Human Capital and Economic Opportunity Working Group Provider-Homepage: http://www. hceconomics. org/ Working Papers*.

Keane, M., & Wolpin, K. (2001). The effect of parental transfers and borrowing constraints on educational attainment. *International Economic Review*, *42*(4), 1051–1103.

Krueger, D., & Ludwig, A. (2013). Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous. *The American Economic Review*, *103*(3), 496–501.

Lee, S. Y., & Seshadri, A. (2012). Nature and nurture in the transmission of economics status. *University of Wisconsin, Working Paper*.

Lochner, L., & Caucutt, E. (2012). Early and late human capital investments, borrowing constraints, and the family. In

2012 meeting papers.

Lochner, L., & Monge-Naranjo, A. (2008). *The nature of credit constraints and human capital* (Tech. Rep.). National Bureau of Economic Research.

Ludwig, A., Schelkle, T., & Vogel, E. (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics*, *15*(1), 94–107.

Manuelli, R., & Seshadri, A. (2005). *Human capital and the wealth of nations*. March.

Mincer, J. (1958). Investment in human capital and personal income distribution. *The Journal of Political Economy*, *66*(4), 281–302.

Pal, R. (2013). Decomposition of income-related inequality in educational performance: Evidence from india. *Journal of Quantitative Economics*.

Schultz, T. (1961). Investment in human capital. *The American Economic Review*, 1–17.

Stiglitz, J., & Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American economic review*, 393–410.

Todd, P., & Wolpin, K. (2003). On the specification and estimation of the production function for cognitive achievement*. *The Economic Journal*, *113*(485), F3–F33.

Woessmann, L. (2004). *How equal are educational opportunities?: Family background and student achievement in europe and the united states* (Tech. Rep.). IZA Discussion paper series.