A Cost Benefit Analysis of Education Policies

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Abstract

This study, by developing a life-cycle model, seeks to mathematically compute the explicit costs of raising human-capital outcomes for different policy initiatives like income support, financial access, subsidised books, greater educational access and higher return on assets. The study explores two income categories, with different levels of financial access and human-capital investment options and examines a two-generation, seventy periods' life-cycle problem of an altruistic parent valuing the human-capital of his offspring and facing competing challenges of his consumption needs during working life and retirement as well as his dependent child's human capital and consumption requirements. The problem is solved using dynamic programming where life-cycle profiles are generated using simulations.

1 Introduction

Starting from Mincer (1958), Becker (1964) and Ben-Porath (1967), the link between human capital of an individual and his life-cycle income is well documented in literature. In addition, there is also evidence of a strong correlation between the child's educational attainment and parental education. While it is partly attributed to heritability of traits i.e. the child inheriting the ability, personality and preferences of parents, another important reason is human capital acquisition i.e. educated parents due to their higher wealth tend to invest more in their child.

Given the link between human capital and income, parents invest in their child's human capital either for altruistic or for exchange motives. Presence of borrowing constraints, however, may force parents to under-invest in their child's education. The child's education attainment, (human capital) , then not only becomes a function of his ability, but also parental altruism, parental income and borrowing constraints.

One extreme of the spectrum is when parents, instead of investing in their children, overdraw from their future by sending them to work. The incidence of child-labour is very high in the third world countries. According to the Bureau of Statistics, International Labour Organization, in 2012 around 168 million children were engaged in child-labour. Half of them were engaged in hazardous work. A number of studies like Pal (2013) using IHDS data set, Filmer and Pritchett (2001) using household asset variables on Indian data and Hill and Duncan (1987) using PSID data, find significant effects of income/wealth on children's education. Basu and Van (1998) argue that only under extreme conditions of very low incomes, parents send their wards to work. Child-labour therefore, is a means of lifting consumption to the bare minimum subsistence level.

Apart from income level, transitory income shocks have also been reported to have an effect on child-labour and education. Kazianga (2005) found that school enrolments in Burkina Faso dropped with higher anticipated transitory income shocks. Similarly, Beegle et al (2006), with Tanzanian data, found that the incidence of child labour increased with higher transitory shocks but also reported that the effect diminished with higher wealth holdings. Other empirical studies have similarly reported fall in enrolment with higher wage shocks. On the other hand, Dehejia and Gatti (2005), and Beegle et al (2006) found evidence of financial development having significant effect in reducing child-labour.

The role of education subsidies in enhancing school enrolment is also widely reported in literature.

Kremer et al (2002), Kremer et al (2009) and Grogan (2009) are some of the studies which reported the positive effects of free uniforms, text-books, scholarships and fee exemptions on schooling.

Since a child does not make his/her human-capital decision, the role of the parent in the human-capital acquisition of a child becomes crucial. Children's education is therefore a household decision problem. There exists significant economics literature on the intergenerational set-up & human capital acquisition. The work of Todd and Wolpin (2003), Cunha and Heckman (2008), Cunha et al (2010), Keane and Wolpin (2001), Johnson (2010) and Del Boca et al (2010) have focussed on the effect of inputs by family along with other environmental conditions which have played a role in shaping a child.

Similarly, Ellwood et al (2000), Carneiro and Heckman (2002), Belley and Lochner (2007), Lochner and Monge-Naranjo (2008) have linked lower college enrolment to borrowing constraints in students from low income families.

While most of the inter-generational literature has looked at human-capital acquisition under different scenarios of heritability of traits, parental investments and borrowing constraints, very few studies have so far focussed on replicating the empirical findings on child-labour and school enrolment through an inter-generational model. In particular, the combined role of financial as well as human-capital parameters along with borrowing constraints in the school-enrolment decision of a parent needs to be further explored.

In attempting to do so, this study numerically solves a seventy time-periods' two-generation life-cycle model of an altruistic parent trying to balance his lifetime consumption requirements as well as his dependant offspring's human-capital and consumption needs. The wages and financial asset returns are uncertain and the parent faces bequests, pension savings and borrowing constraints.

In the developing world, a large number of households continue to remain financially excluded, and often do not have access to good schools. In order to take these inequalities into account, and also to segregate the marginal effects of financial access and human-capital access on consumption and education, this study explores two income categories with different levels of financial and human-capital investment options. The study compares the costs of different policies like education subsidy, income support, asset returns and financial access in raising human-capital investments of households.

2 The Environment

The model assumes a single parent (dad) with a single offspring (son) both of whom would be jointly referred to as the household.

Time is discrete and the life-cycle model starts from time, $t = 1$, when the son is just an infant (age=1) and the dad has started working (age=21).
"t" denotes absolute time in years, "age" denotes the age in years of the person in focus (either father or son). For the son, t and age will be the same. There are 70 time periods in the model.
A phase is defined as the son's phase in life: education phase and adult phase. The first 20 time periods ($1 \leq t \leq 20$) have been defined as the education phase. Adult phase starts from $t = 21$ onwards. The education phase is also when the dad and son are a joint household. Hence, the terms education phase and joint phase are used interchangeably.

In the first five years (pre-school stage) there are neither education nor labour options available to the son. The only expense the dad incurs on the son, during this period, is consumption expense. Even though a child consumes less than the parent, in the simulations the dad-son consumption expenditures are assumed as equal since health related expenses of the child are not explicitly modelled.

From $6 \leq age \leq 20$ the son can either go to acquire human-capital or work in the informal child-labour market to augment the family income. (Note: A definition of a child is someone who is not an adult). The time of entry of the independent (grown-up) son into the formal labour market is denoted as $t = sw = 21$ (son working) which is also the time the son separates and starts a new family.

From $t = sw = 21$ onwards, the dad and son are economically separate and have no interaction except for a last period (of dad's life) bequest. At time $t = fr = 41$ the father retires (father's age=61) and the father is dead at time $t = fd = 50$ (father's age=70). He leaves a pre-determined bequest, which is a constant fraction of his last-period wealth, to his son.

The son is assumed to start his adult life with zero financial assets. The son retires at $t = sr = 61$ and dies at $t = sd = 70$ which is also the last period of the model. The model stays away from a dynastic set-up by including further generations.

Human-capital

From, $6 \leq t \leq 20$, the dad has an option of either educating the son or sending him to the child-labour market or a combination of both. The son is endowed with a unit amount of time in each period. The fraction of time devoted to education in time-period t is denoted as λ_t. The dad also decides the amount of books b_t to be invested in each period. The son's future income as an adult depends on the level of human capital accumulated in the education phase ($6 \leq t \leq 20$).

The household faces borrowing constraints every period as the dad cannot borrow against his son's future labour income due to moral hazard problems. The model also assumes that the dad has no borrowing options against his own future labour income due to labour-income risk.

The father's human capital $H_{f,t}$[1] $= H_{dad}$ is assumed constant over time. The Ben-Porath (1967) model was used for the human capital production function. The child's human capital at period $t+1$, in the education phase, is the function of previous period's human capital $H_{s,t}$, ability level θ, time fraction spent studying λ_t, books invested b_t and the rate of depreciation δ.

$$\begin{aligned} H_{f,t+1} &= H_{f,t} = H_{dad} \\ H_{s,t+1} = f(\theta, \lambda_t, H_{s,t}, b_t) &= H_{s,t}(1-\delta) + \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} \end{aligned} \tag{1}$$

The human capital of the pre-school son is assumed to stay constant and is assumed as equal to the base level of human capital $\bar{H}$(which is the lowest possible level of human-capital and there is no further depreciation from that). Human-capital acquisition can start from age 6 onwards.
Since, the nutritional effect on human-capital is not modelled in the pre-school stage, human-capital is assumed to remain at the base level upto age 6. Also, since the dad is solving the life-cycle model at $t = 1$ and assumes the son will be a separate household once he reaches age 21, the dad only takes into account formal education upto graduation. Human-capital accumulation prospects of the independent son through On-the-Job Training (OJT) is not modelled.

At any time period t, the maximum human-capital the child can enter the next time-period with, corresponds to $\lambda_t = \lambda_{max}$ and $b_t = b_{max}$.

$$H_{s,t+1}(max) = f(\theta, \lambda_{max}, H_{s,t}, b_{max}) = H_{s,t}(1-\delta) + \theta(\lambda_{max})^{aa}(H_{s,t})^{bb}(b_{max})^{cc} \tag{2}$$

$\lambda_{max} = 0$ when the child is in the pre-school stage and equal to 1 for other periods in the education phase. b_{max} is the maximum investment possible in books. b_{max} is zero at pre-school and then constant for a given income category. It varies for different income categories to reflect the inaccessibility of good schools/educational inputs for the poor households.

The dad has an explicit per-period utility from the educational inputs (education-time and books) he is able to afford for his child's human-capital production. The utility has the following functional form:

$$u(\lambda_t, b_t) = pen.\Big(H_{s,t+1}(\lambda_t, b_t) - H_{s,t+1}(\lambda_{max}, b_{max})\Big) \tag{3}$$

[1]subscript f stands for father, s stands for son

where *pen* is a constant number signifying the amount of utility gain per-unit of additional human capital produced. *pen* is calibrated in the model such that a father with matriculation level of education, on average (10000 simulations), teaches his son upto matriculation.

Thus $u(0,0) = -pen.H_{s,t+1}(max)$ and $u(\lambda_{max}, b_{max}) = 0$ and u is an increasing function of the inputs with an upper bound of zero.

Father's wage
Wages are subjected to a per period wage shock e_t which can take two values, $e_t = \{e_{low}, e_{high}\}$ such that $\mathbf{E}[e_t] = 1$. The actual wage of an adult with human-capital level H_{dad} at time t for given wage shock realization e_t

$$WAGE(H_{dad}, t, e_t) = \bar{G}^{t-1} H_{dad} e_t \tag{4}$$

where $\bar{G}$ is the exogenous growth in expected wage rate every year.

Therefore, the minimum expected adult wage corresponds to that of an adult with base level of human-capital ($\bar{H}$) and is equal to one[2] at $t = 1$ and it grows exogenously at growth rate $\bar{G}$. In monetary terms 1 would be equivalent to Rs 36,000 per year (Rs 100/day).

Child-labour wages
Since child-labor is not a formal market activity, the wages are not a function of the human-capital level of the child. Since mostly it is physical labor, it is assumed to be a function of the age bracket the child falls into and hence, some fraction of the minimum adult wage.
At any given time t, for human-capital of son $H_{s,t}$, the child-labour wage per unit-time (CLWAGE) is equal to :

$$CLWAGE(H_{s,t}, t) = \bar{G}^{t-1} H_{relv} \tag{5}$$

where H_{relv} is the relevant, age dependent, child-labour wage rate defined in Table 1

Table 1: Child-labour wage rates

	$0 \le age \le 5$	$6 \le age \le 10$	$11 \le age \le 15$	$16 \le age \le 20\}$
H_{relv}	0	$0.33H$	$0.5H$	$H_{relv} = H_{matric}$ if $H_{s,t} > H_{matric}$ else $H_{relv} = \bar{H} = 1$

H_{matric} is the matriculation level of human-capital and $\bar{H}$ is the base level of human-capital.
A child investing λ_t fraction of time in education would earn $\bar{G}^{t-1} H_{relv}(1 - \lambda_t)$.

The wage rate of the child between $6 \le age \le 10$ is 33% of the minimum adult wage rate. The wage rate of the son between $11 \le age \le 15$ is 50% of the minimum adult wage rate. From $16 \le age \le 20$ the wage rate is 100% of the minimum adult wage rate if the human-capital level of the son is less than H_{matric} or else its matriculation wage rate (which is 200% of the minimum adult wage rate).

For ease of computation, a simplifying assumption is made that the child's labour-wage is not subject to wage shocks. Since, the child-labour wage is anyway a fraction of the minimum adult wage the assumption would not result in major discrepancies in net-household income. Also, since child-labour is a non-market interaction, it need not be completely subjected to labour-market shocks and is anyway not priced as a function of actual human-capital.

Financial assets
Financial assets consist of a risk-less one-period bond of gross real return R_0 and a risky one-period stock

[2] $\bar{H}$ is calibrated to be equal to 1.0

with gross real return R_{γ_t} subject to a per period stock shock.

$$R_{\gamma_t} = R_0 + \mu + \gamma_t \tag{6}$$

γ_t is the time t shock to the stock return which can take two values. $\gamma_t = \{\gamma_{low}, \gamma_{high}\}$ such that $\mathbf{E}[\gamma_t] = 0$. μ is the excess return of stock over bond. Shocks to labour-wage e_t and stock returns γ_t can be correlated.

Based on the father's level of human capital, he is assumed to either have no financial access, only bond market access or full financial access. A father with no financial access would have negative real returns on savings whereas a father with only bond access will not have the option of investing in stock. Only a father with full financial access would have a savings portfolio consisting of a bond and a stock.

Since solving the life-cycle portfolio choice of the highest income category is one of the objectives of this study, a full seventy period life-cycle model becomes necessary instead of a condensed version with several time-periods clubbed together for faster computation.

Expected Utility Maximization

Let $\Omega_t = (\gamma_t, e_t)$ denote the vector of the wage shock e_t and stock market shock γ_t at any time t. Both e_t and γ_t are individually uncorrelated over time. However, they can be cross-sectionally correlated. Since, both wage and stock shock can take 2 states each, Ω_t can take 4 possible states $\Big((\gamma_{low}, e_{low}), (\gamma_{low}, e_{high}), (\gamma_{high}, e_{low}), (\gamma_{high}, e_{high})\Big)$.

$\Omega^{(t)} = \Omega_1, \Omega_2, \Omega_3 ....\Omega_t$ denotes the history of joint shocks[3] upto upto time t.

Utility from consumption has been modelled as a standard CRRA utility,

$$u(C) = C^{1-\xi}/(1-\xi)$$

The father's utility from educational-inputs at time t for education time λ_t and books b_t is:

$$u(\lambda_t, b_t) = pen.\Big(H_{s,t+1}(\lambda_t, b_t) - H_{s,t+1}(\lambda_{max}, b_{max})\Big)$$

At time $t = 1$ and aggregate state Ω_1, for resources-in-hand $X_{f,1}$, human-capital of son $H_{s,1}$ and contributed-pension-fund deposit $PF_{f,1}$, the father seeks to maximize the expected utility from his and his son's lifetime consumption accounting for per-period utility for educational inputs in the education phase. Hence, the father's optimization problem subject to constraints is :

$$\max_{C_{f,t}, C_{s,t}, \lambda_t, \alpha_t, b_t} \Bigg(\Big(u(C_{f,1}) + \rho u(C_{s,1}) + u(\lambda_1, b_1) \Big) + \beta^{t-1} \mathbf{E}\Big[\sum_{t=2}^{20} u(\lambda_t(\Omega^{(t)}), b_t(\Omega^{(t)})) \Big] + \beta^{t-1} \mathbf{E}\Big[\sum_{t=2}^{50} u(C_{f,t}(\Omega^{(t)})) + \rho \Big(\sum_{t=2}^{70} u(C_{s,t}(\Omega^{(t)})) \Big) \Big] \Bigg)$$

[3]It must be noted that $\Omega^{(t)}$ is different from Ω_t (which denotes the realization of the joint shock at time t). An example of $\Omega^{(t)}$ for t=2 would be $\Omega^{(2)} = \Big((\gamma_{low}, e_{high}), (\gamma_{high}, e_{high})\Big)$

subject to :

$$
\begin{aligned}
X_{f,t} + \bar{G}^{t-1} H_{relv}(1-\lambda_t) - s.b_t &= C_{f,t} + C_{s,t} + a'_t \\
X_{f,t+1}(\Omega_{t+1}) &= a'_t R_{p,t+1} + \bar{G}^t e_{t+1} H_{dad}(1-\tau) \\
where\Big(R_{p,t+1} &= R_0 + \alpha_t(R_{\gamma_{t+1}} - R_0)\Big) \\
H_{s,t+1} = f(\theta, \lambda_t, H_{s,t}, b_t) &= H_{s,t}(1-\delta) + \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} \\
PF_{f,t+1}(\Omega_{t+1}) &= PF_{f,t}R_0 + \bar{G}^t H_{dad} e_{t+1}\tau \\
a'_t \geq 0, \quad 0 \leq \alpha_t &\leq 1, \quad 0 \leq \lambda_t \leq 1, \quad b_t \leq b_{max}
\end{aligned}
$$

$\Omega^{(t)} = \Omega_2, \Omega_3....\Omega_t$ is the history at time t,
$C_{f,t}(\Omega^{(t)})$: father's consumption at time t for history $\Omega^{(t)}$,
$C_{s,t}(\Omega^{(t)})$: son's consumption at time t for history $\Omega^{(t)}$,
$\lambda_t(\Omega^{(t)})$: fraction of education time at time t for history $\Omega^{(t)}$,
$b_t(\Omega^{(t)})$: investment in books at time t for history $\Omega^{(t)}$,
ρ: downward altruism coefficient, is equal to 1 upto $t = 20$ and then some constant value lower than one for rest of the time-periods.
s: fraction of the books b_t the household is required to pay
a'_t: savings at time t
$PF_{f,t}$: current level of father's pension fund level at time t.

A detailed review of the solution methodology and constraints is included in the Appendix.

The grown-up son's future consumption and utility from consumption will depend on his accumulated human capital level in the first 20 periods and the father cares about it through downward altruism. Beyond the utility from consumption that human-capital affords, the father explicitly values the acquisition of human-capital which the model seeks to capture through the utility from educational inputs term $u(\lambda_t, b_t)$.

The father solves the problem at $t = 1$ taking into account that his independent son will have to take care of his life-time consumption requirements. Hence, the income from the endogenous human-capital the father leaves the adult son with will be used to meet the consumption requirements of the son. The model stays away from a dynastic setup by not including future generations into the life-cycle optimization.

3 Benchmark Parameters

3.1 The Two Benchmark Income Levels

The primary aim of this study is to quantify the marginal impacts of various policy parameters like income support, financial inclusion, bond returns, educational subsidies, access to education and others on per-period consumption, savings and human-capital investments. Since the impact on consumption and human-capital investments will be different for different income brackets this study takes three benchmark education levels of the parent for analysis :

1. The Uneducated Dad $\Big(H_{dad} = 1.0\Big)$

2. The Matric Dad $\Big(H_{dad} = 2.0\Big)$

This study assumes a linear relationship between expected income and human-capital level, therefore, the empirically reported ratios of incomes for different education levels have been used to calibrate human-capital levels. As per existing studies, the average income level of an individual with matriculation is roughly twice that of an illiterate person. Hence, the human-capital level of the matric dad is assumed as twice of an uneducated dad. The graduate dad's human-capital level was calculated based on empirically reported average returns (to human-capital) of 11% for an individual with graduation level of education.

It needs to be emphasized that actual income variation in the population is much higher than five times due to a host of factors like permanent shocks, experience levels, OJT etc. However, the average income of all individuals with graduation level of education is roughly five times the average income of all individuals with no-education (as is empirically reported).

This model only considers transitory income shocks for analysis. Including permanent shocks would slightly improve the quality of results by capturing the additional impact of shock persistence over time. The impact of shocks on life-cycle parameters is one of the many parameters of interest for this study and hence, this study keeps the set-up simple by including transitory only shocks.

Pension-fund savings are ignored in the benchmark scenarios but will be considered later in the parametric analysis.

3.1.1 The Uneducated Dad

The uneducated dad, with zero years of schooling, is assumed to have the base level of human-capital, $\bar{H} = 1$. Therefore, his income at any given time t is $\bar{G}^{t-1}\bar{H}e_t$ (where e_t is the wage shock). The household is financially excluded and the savings yield negative real returns.
The household gets a 50% discount on books and the maximum investment the household can make on books in any period is capped at $b_{max} = 0.25$. The cap on investment is to reflect the scenario of limited access to good schools or other education aids based on income criterion. Pension-fund savings, are not applicable for this income group.

3.1.2 The Matric Dad

The matric dad is assumed to have a human-capital level $H_{matric} = 2$ and limited financial access (only bond market access). The maximum investment this household can make on books in any period is capped at $b_{max} = 0.5$.

The summary of the benchmark scenarios is presented in Table 2. In all the three scenarios the son's value function is computed with the assumption of bond market access to the son so as to keep it uniform. Since the focus is on the father's life-cycle decision problem, the attempt is to ensure that differences in son's life-cycle values should arise only out of differences in human-capital levels.

Table 2: The three benchmark scenarios

	H_{dad}	Fin Access	Books-Discount	Book-Max	Pension-Fund
UnEduDad	1.0	None	50%	0.25	NA
MatricDad	2.0	Bond	50%	0.5	0

3.2 Consumption and Human Capital Parameters

The utility from consumption has been modelled as a standard CRRA utility,

$$u(C) = C^{1-\xi}/(1-\xi) \tag{7}$$

with the assumption of a moderately risk averse individual with $\xi = 5$.

The Ben-Porath (1967) model was used for the human capital production function, where:

$$\begin{aligned} H_{s,t+1} = (1-\delta)H_{s,t} + \theta(\lambda_t)^{aa}(H_{s,t})^{bb}(b_t)^{cc} \qquad & 6 \leq \mathbf{t} \leq 20 \\ H_{s,t+1} = H_{s,t} = \bar{H} = 1 \qquad & \mathbf{t} \ < 6 \end{aligned} \tag{8}$$

$\bar{H}$ is the base level of human-capital which is normalized to 1. The minimum adult expected wage corresponds to that of an adult with base level of human-capital. It is equal to 1 at $t = 1$[4]. In monetary terms 1 would be equivalent to Rs 36,000 per year (Rs 100/day).

Ability, θ and the other human-capital production coefficients, *aa*, *bb*, *cc* were used from Manuelli and Seshadri (2005) with minor modifications to give an average return of 11% when the child studies full-time with the maximum available books, $b_{max} = 1$. The father's downward altruism coefficient ρ is assumed to be 1 in the joint phase and 0.7 for the rest of the independent son's life-cycle. As discussed in the previous section, even though the dependent son would normally consume less than the father, we assume father-son consumption expenditures as equal ($\rho = 1$) in the joint phase since we are not explicitly accounting for health and other expenses of the child.

Several studies in literature have reported high correlations between the parent and the child's educational attainment[5]. The parameter *pen*, which is a constant number capturing the per-period father's utility from unit gain in son's human-capital, was calibrated to ensure that on average over 10,000 simulations a father with matriculation level of human-capital ($H_{dad} = 2$) would teach his son upto the matriculation level. After, calibration it was set to $pen = 1.71$.

Table 3: Benchmark : Consumption and human capital parameters

ξ	θ	aa(=bb)	cc	δ	ρ	discount	pen	H	H_{matric}
5	0.16	0.75	0.2	1.8%	1/0.7	50%	2.55	1	2

3.3 Financial Parameters

A bond rate of $r_0 = 4\%$[6] is assumed and expected stock returns are assumed at $\mathbf{E}(r_s) = 9\%$[7]. The returns for the financially excluded uneducated dad household is assumed as $r_{nofin} = -5\%$. The variance of innovation to risky asset (stock shock) is taken as $var(\gamma_t) = 0.0246$ and the variance of logarithm of transitory wage shocks, $log(e_t)$, has been calibrated at 0.085[8]. (The values of stock and wage shock variance are close to the values in Cocco et al (2005)) The correlation between income shocks and stock returns, has been assumed as $corr = 0$.

Discount rate is set as $\beta = 0.98$. The father leaves 15% of his last period's wealth as planned bequest to his son. Contributed-pension-fund deposits, if applicable, earn bond interest rates and are risk-free. However in the benchmark case the deduction rate τ is assumed zero.
The exogenous growth rate of income is assumed at $\bar{G} = 1\%$.

Table 4: Benchmark : Financial parameters

r_0	$\mathbf{E}r_s$	r_{nofin}	var(γ_t)	var($log(e_t)$)	corr	β	τ	Bequest	G
4%	9%	−5%	0.0246	0.085	0	0.98	0%	15%	1%

4 Simulation Results : Two Benchmark Scenarios

This section presents the simulation results for the three benchmark scenarios mentioned in the previous section. The father's affinity for human-capital parameter (*pen*) was calibrated such that on average the matric dad teaches his son upto matric or higher for 75% of the trials. In order to generate average consumption and human-capital investment profiles repeated simulations trials were done by drawing numerous realizations of the income and stock return shocks. The life-cycle profile values reported here

[4]it grows exogenously at growth rate $\bar{G}$

[5]Ermisch and Francesconi (2001), Woessmann (2004), Black et al (2003)

[6]$R_0 = 1.04$

[7]$\mathbf{E}R_s = 1.09$

[8]where $E[log(e_t)] = 0$

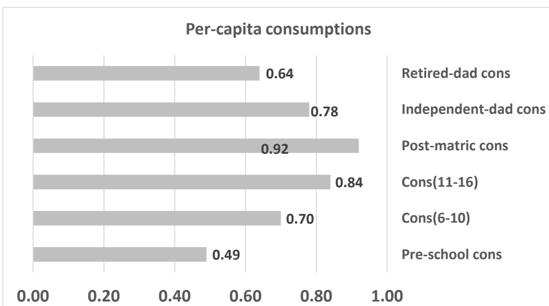
are the cross-section means of 10,000 trials.

The per-capita consumption levels in the joint periods $\left(C_{joint(1-5)}{}^{9},\ C_{joint(6-10)},\ C_{joint(11-15)}\right.$ and $\left.C_{joint(16-20)}\right)$ and independent dad's working and retired period consumptions, $\left(C_{dad(21-40)}\right.$ and $\left.C_{dad(41-50)}\right)$ as well as educational investments of the three households are presented in Table 5. Figure 1 plots the per-capita consumption levels of the two households. The rationale for breaking up the 20 joint periods into 4 sub-periods (1-5, 6-10, 11-15, 16-20) is the applicable child-labour wage rate for the corresponding time block.

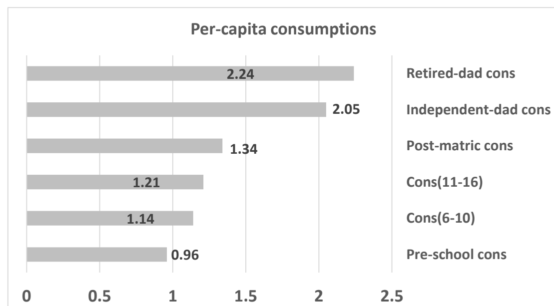
Table 5: Benchmark values of bequest, net education-time, net books and human-capital level of the son

	Beq	$Edu(yrs)$	$Books(Tot)$	H_{son}
UnEduDad	0.11	0	0	1.00
MatricDad	0.40	8.74	4.18	2.41

1.0 consumption unit is equivalent to \$2/day. Edu(years) & Books(Tot) refers to total investment in education-time and books for 15 years ($6 \leq t \leq 20$) in the education-phase.



1.1 Unedu dad consumptions

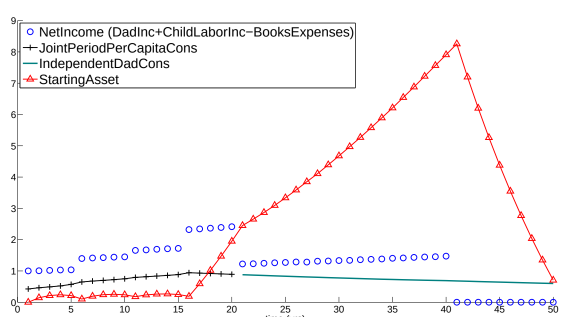


1.2 Matric dad consumptions

Figure 1: Per-capita consumption values for 20 joint-periods & 30 independent & retired dad's periods. 1.0 is equivalent to \$2/day.

Uneducated Dad ($H_{dad} = 1$): Figure 2 depicts the income, per-capita consumption and the starting assets for the 50 time-periods of the dad's life. The first 20 periods are the joint-periods and the next 30 time-periods are the independent dad's periods.

The simulation output reveals that child-labour is an optimal outcome of the life-cycle model. The son is never sent to school and the investment in books is zero.



2.1 Uneducated Dad

Figure 2: Uneducated dad's life-cycle profiles

[9]refers to mean per-capita consumption for the time interval $1 \leq t \leq 5$

Table 6: Uneducated dad life-cycle data

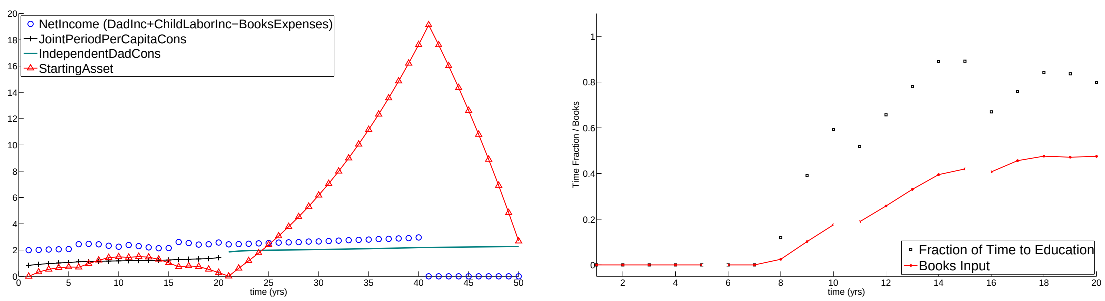
	NI	PV(NI)	PV(NIjnt)	NS	PV(NS)	PV(NSjnt)	TB	PV(TB)
Base	63.47	31.04	19.94	169.96	47.5	5.69	0.4659	0.228
Fin	63.47	31.04	19.94	176.76	47.56	6.94	0.55	0.26

Tax-rate: pension fund deduction rate; NI (Net Income) : sum of 50 periods' income; NIjnt: sum of twenty joint-periods' income; NS (Net Savings) : sum (for all 50 periods) of each periods' bond market savings; NSjnt: total savings in twenty joint-periods; TB (Total books) : sum of total books invested; '-' : data not relevant. PV refers to present value discounted at bond rate.

Because the household is assumed to be financially excluded (negative real returns), the life-cycle consumption trend is expected to be downward sloping ($\beta(1+R) < 1$). However, due to increasing child-labour wage rate slabs and strict borrowing constraints, early period consumption is upward sloping resulting in a hump shaped life-cycle consumption profile for the dad. Even though net family income is wavy, consumption follows a smooth trend. The assets in the joint-periods are close to zero, revealing that the net income is entirely spent on consumption. Thereafter, the assets follow the usual humped shape peaking the year before the dad's retirement.

Overall, the simulation results reveal that without external intervention, this would be a self-perpetuating inter-generational poverty trap where the child's future gets traded-off and the son, like father, starts his adulthood with no education.

Matric Dad (*$H_{dad} = 2$*): At this income level, per-capita consumption is upward sloping. The



(a) Net-Income, Consumption and Assets

(b) Human-capital investments (education-time & books)

Figure 3: Matric dad's life-cycle profiles

independent dad's average consumption levels $\left(C_{dad(21-40)} \text{ and } C_{dad(41-50)}\right)$[10] owing to son's expenses no longer being there witnesses a jump at $t = 21$ leading to a discontinuity in the consumption trend.

Figure 3 shows the educational inputs (education time and books) invested in the son's human-

Table 7: Matric dad life-cycle data

	NI	PV(NI)	PV(NIjnt)	NS	PV(NS)	PV(NSjnt)	TB	PV(TB)
Base	97.68	47.79	30.7	269.6	73.7	12.21	4.18	2.37
Fin	97.68	47.79	30.7	322.53	88.93	17.50	4.47	2.54

Tax-rate: pension fund deduction rate; NI (Net Income) : sum of 50 periods' income; NIjnt: sum of twenty joint-periods' income; NS (Net Savings) : sum (for all 50 periods) of each periods' bond market savings; NSjnt: total savings in twenty joint-periods; TB (Total books) : sum of total books invested; '-' : data not relevant. PV refers to present value discounted at bond rate.

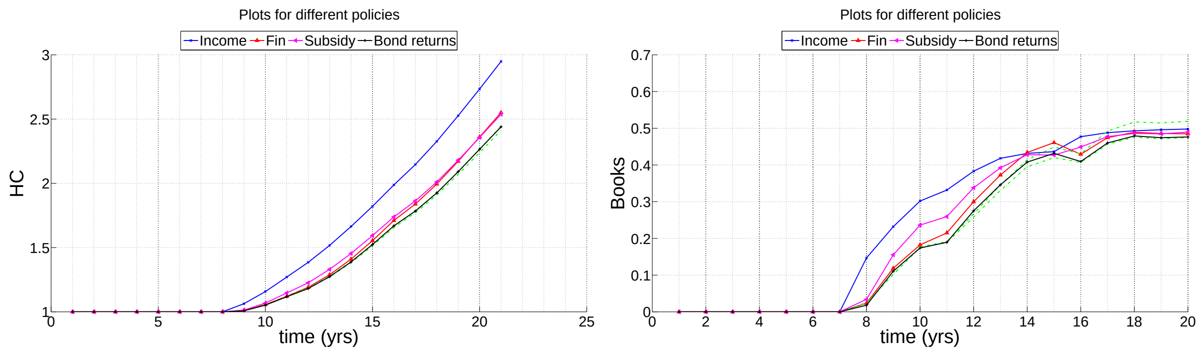
capital. The son gets partial education during all periods. In the initial periods when the assets are low, the fraction of time devoted to education is lower but it increases gradually till upto $t = 10$. From $t = 11$, the opportunity cost of education increases owing to a higher child-labour wage rates, leading to a

[10] $21 \leq t \leq 50$

reconfiguration of investment in time and books. A similar trend gets repeated from $t = 16$ when again, the child-wage rates change.

5 Cost of Policies

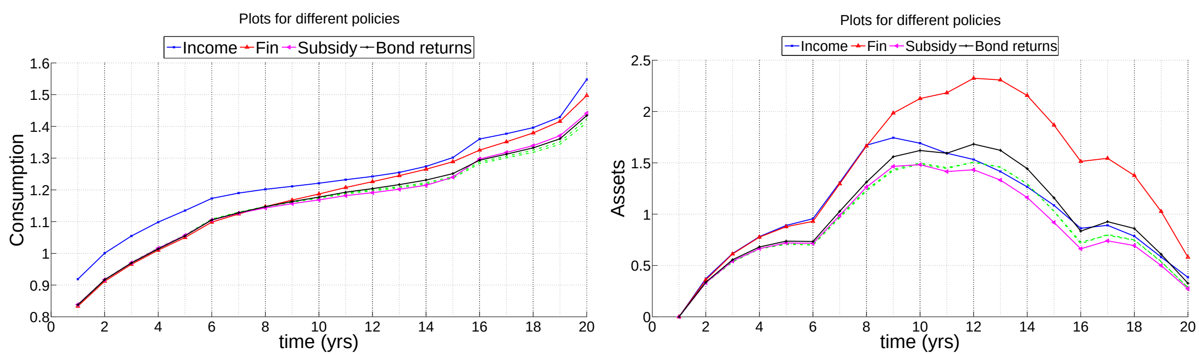
5.1 Matric dad



4.1 Human-capital

4.2 Books

Figure 4: Matric dad : human-capital and books' profiles for different policies



5.1 Consumption

5.2 Assets

Figure 5: Matric dad : consumption and assets' profiles for different policies

5.1.1 Dad's Income

Policy-maker's cost of human-capital At risk-free discounting rate, the present value of the dad's fifty periods' income amounts to 47.79 units and present value of twenty joint-periods' income is 30.7 units (Table 7). Since the dad doesn't carry forward assets from joint-phase into the independent phase, the independent periods' incomes are delinked from human-capital investments and consumption in the joint-phase. With elasticity of human-capital with income at around 2.23, one consumption unit in present value terms raises human-capital of the son by 7.26%[11]. Extra income funded by the policy-maker, though effective, turns out to be a costly way of raising human-capital for this income category as opposed to other policies like subsidized books or greater educational access.

[11] =2.23/0.307

Table 8: Matric dad: Percentage changes from benchmark values for different policies

	H_{son}	$Edu(yrs)$	$Books(Tot)$	C_{joint}
Income	22.3%	28.37%	22.78%	5.74%
Fin	5.77%	6.38%	6.94%	1.77%
Books-Subsidy	5.23%	5.34%	11.38%	0.04%
Books-Access	1.32%	0.52%	6.28%	-0.28%
Bond Returns	1.19%	1.23%	1.70%	0.25%

Edu(years) & Books(Tot) refers to total investment in education-time and books for 15 years ($6 \leq t \leq 20$) in the education-phase.

5.1.2 Financial Access

Policy-maker's cost of human-capital Without incurring any explicit cost of giving stock market access to the matric dad, human-capital can be raised by 5.77% which, in strict human-capital outcome terms, is equivalent to 2.58% permanent income increase[12] of the dad.

5.1.3 Bond Returns

Policy-maker's cost of human-capital The matric dad's investments in the bond market in all fifty time-periods amounts to 270 units which in present value terms amounts to 73 units (Table 7). The present value of his savings in the twenty joint periods amounts to 12.21 units. With elasticity of human-capital with bond returns at 1.19, a unit of present consumption leads to 9.75%[13] rise in human-capital. Assuming the extra return on bonds are completely funded by the policy-maker, compared to income support additional bond returns are only 134% as effective in raising human-capital with the same cost in present-value terms.

5.1.4 Books' Subsidy

Policy-maker's cost of human-capital The total investment in books for the matric dad household is 4.18 units with present value of 2.37 units (Table 7). With elasticity of human-capital for additional discount-percentage at around 0.52, one unit of consumption in present value terms increases human-capital by 21.9%[14]. Books' subsidy therefore is 2.3 times more effective in raising human-capital than income support.

5.1.5 Higher Access to Books

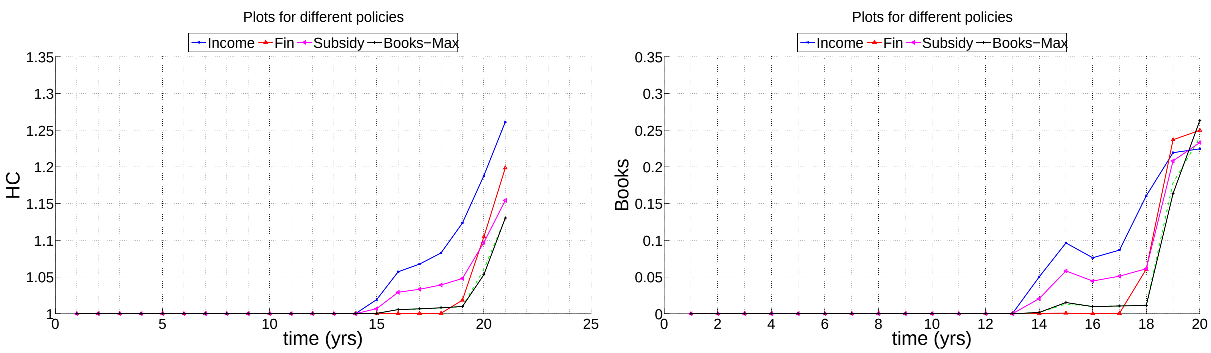
Policy-maker's cost of human-capital The total investment in books for the matric dad household is 4.18 units with present value of 2.37 units (Table 7). 1% higher access to books costs 0.0237/2 units of consumption in present value terms to the policy maker (with 50% discount borne by the policy-maker). With elasticity of human-capital for additional books' access at around 0.132, one unit of consumption in present value terms increases human-capital by 11.18%[15]. Books' subsidy therefore is 1.38 times more effective in raising human-capital than income support.

5.2 Uneducated dad

5.2.1 Dad's Income

Policy-maker's cost of human-capital At risk-free discounting rate, the present value of the dad's fifty periods' income amounts to 31.04 units (Table 6). With elasticity of human-capital with income at around 1.16, one consumption unit in present value terms raises human-capital of the son by 3.73%[16].

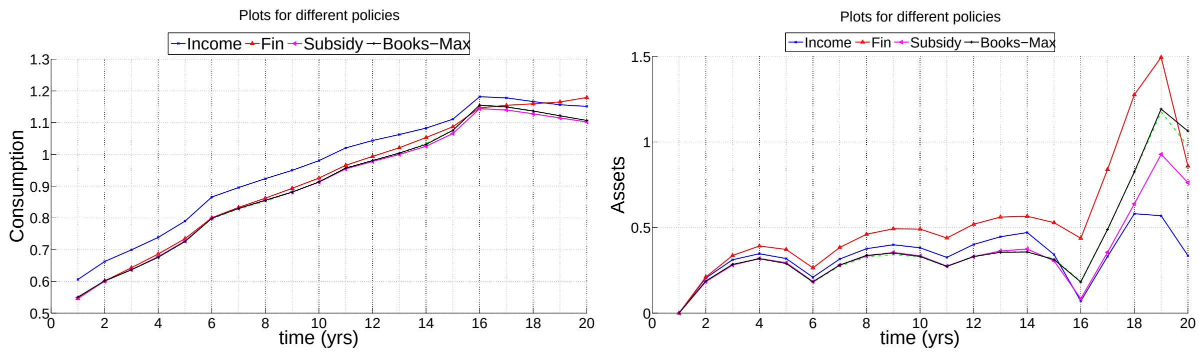
[12]=5.77/2.23
[13]1.19/0.1221
[14]0.52/0.0237
[15]0.132/0.0118
[16]=1.16/0.3104



6.1 Human-capital

6.2 Books

Figure 6: Uneducated dad : human-capital and books' profiles for different policies



7.1 Consumption

7.2 Assets

Figure 7: Uneducated dad : consumption and assets' profiles for different policies

5.2.2 Financial Access

Policy-maker's cost of human-capital Without incurring any explicit cost of giving bond market access to the uneducated dad, human-capital can be raised by 6.06% which, in strict human-capital outcome terms, is equivalent to 5.22% permanent income increase[17] of the dad.

5.2.3 Books' Subsidy

Policy-maker's cost of human-capital The total investment in books for the uneducated dad household is 0.465 units with present value of 0.228 units (Table 6). With elasticity of human-capital for additional discount-percentage at around 0.21, one unit of consumption in present value terms increases human-capital by 92.1%[18]. Books' subsidy therefore is 24.6 times more effective in raising human-capital than income support.

5.2.4 Higher Access to Books

Policy-maker's cost of human-capital The total investment in books for the matric dad household is 0.465 units with present value of 0.228 units (Table 6). 1% higher access to books costs 0.00228/2 units of consumption in present value terms to the policy maker (with 50% discount borne by the policy-maker). With elasticity of human-capital for additional books' access at around 0.002, one unit of consumption in present value terms increases human-capital by 1.42%[19]. Books' subsidy therefore is 0.38 times as

[17]=6.06/1.16
[18]0.21/0.00228
[19]0.002/0.0014

Table 9: Uneducated dad: Percentage changes from benchmark values for different policies

	H_{son}	$Edu(yrs)$	$Books(Tot)$	C_{joint}
Income	11.61%	99.96%	96.11%	5.95%
Fin	6.06%	49.37%	17.97%	1.48%
Books-Subsidy	2.15%	23.41%	45.28%	1.48%
Books-Access	0.02%	-0.78%	2.02%	0.02%

Edu(years) & Books(Tot) refers to total investment in education-time and books for 15 years ($6 \leq t \leq 20$) in the education-phase.

effective in raising human-capital than income support.

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