Shared Capacity Routing Problem for Buy-Online-Pickup-in-Store Order Fulfillment

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Abstract

To thrive in the competitive market, retailers are increasingly adopting omnichannel strategies to provide a seamless customer experience. One popular model within omnichannel retailing is Buy-Online-Pickup-in-Store (BOPS), which has gained significant popularity in recent years. The primary aim of this thesis is to propose a comprehensive and practical optimization model that effectively reduces the delivery distance for BOPS order fulfillment.

BOPS orders (PUP orders, hereafter) are typically delivered to pick-up points (PUPs), which are usually traditional retail stores. Customers can place their orders online and collect them from designated PUPs. The fulfillment of PUP orders is typically handled by dedicated warehouses. Big retailers like Walmart and Tesco utilize dedicated warehouses to fulfill online PUP orders.

This thesis explores the challenges associated with the joint planning of routes for store replenishment and PUP orders. Currently, two vehicles are used daily: one replenishes the store inventory, while the other delivers PUP orders. However, coordinating the planning of these routes is a challenging task due to operational constraints. Replenishment routes are typically planned well in advance, while PUP routes are planned later due to their shorter lead time.

To optimize the delivery process and minimize the total distance traveled for PUP orders, we propose the concept of capacity sharing. Capacity sharing involves utilizing the spare vehicle capacity of the replenishment vehicles by piggybacking PUP orders to transport them. This can be achieved by transferring the PUP orders onto the replenishment vehicles

at the retail stores or the replenishment warehouse, depending on the availability of spare capacity in both the vehicles and the retail stores.

A problem variant has been previously characterized by Paul, Agatz, Spliet, and Koster (2019) and Paul, Agatz, and Savelsbergh (2019). Paul, Agatz, Spliet, and Koster (2019) introduced a capacity sharing model with replenishment warehouse as a transfer location, which results in low distance savings. To overcome this, Paul, Agatz, and Savelsbergh (2019) proposed using retail stores as transfer locations, focusing on a simplified scenario. This thesis addresses more complex real-life situations, exploring the shared capacity routing problem with multiple routes and considering vehicle capacity constraints for comprehensive optimization in PUP order delivery.

This thesis addresses the following research questions to investigate the feasibility and benefits of implementing capacity sharing for replenishment and PUP route planning.

RQ1. How can capacity sharing between replenishment and PUP routes be optimized to minimize the total distance of BOPS delivery, and what are the associated benefits?

RQ2. What is the impact of operational parameters, such as PUP store overlap, PUP demand size, retail store capacity, vehicle spare capacity, early start of PUP route, and transfer costs, on the distance savings resulting from capacity sharing between replenishment and PUP routes?

We introduce a novel approach to the vehicle routing problem of fulfilling PUP orders. To address this problem, we have developed a mixed-integer linear programming (MILP) formulation tailored to our specific problem context. We present analytical results for smaller simulated instances while proposing efficient heuristics to handle larger instances. To validate the effectiveness of our heuristics, we have conducted a thorough comparison and benchmarking against optimal results. Additionally, we have identified the advantages of our proposed model in various realistic scenarios.

This study reveals that adopting capacity sharing reduces the delivery distance for PUP orders significantly, demonstrating a notable reduction in total route distance compared to not implementing capacity sharing. This minimizes delivery costs and brings environmental benefits by reducing greenhouse gas (GHG) or carbon emissions.

When the replenishment and PUP warehouses are located together, the distance savings are even greater than when they are not co-located. However, the distance savings decrease as the relative size of PUP demand, the number of capacitated retail stores, and transfer costs increase. On the other hand, the distance savings increase when there is more spare capacity in the replenishment vehicles at the warehouse and when the PUP route starts earlier than the replenishment routes.

It is noteworthy that the number of PUP vehicles necessary for capacity sharing is always equal to or greater than the number of PUP vehicles required in the absence of capacity sharing. The distance savings are higher when the number of PUP routes matches the number of replenishment routes.

The present study offers a generalized distance-saving model for PUP order delivery with significant managerial implications. Managers can benefit from optimal route planning, transfer location optimization, efficient store selection, and overall cost reduction by leveraging the insights provided by our model, leading to improved efficiency, cost-effectiveness, and performance in PUP order fulfillment processes.

However, it is noteworthy that the present thesis is not free from limitations. The model lacks real-life industry validation, relying on simulated data to demonstrate significant savings. Additionally, the model overlooks factors such as capital costs associated with using PUP vehicles and loading/unloading times at transfer locations. Further research opportunities include investigating partial deliveries through direct visits and transfers, allowing multiple visits to retail stores, considering a heterogeneous fleet, optimizing replenishment and PUP routes simultaneously, incorporating pure PUP stores, considering product types and density, adopting a multi-objective approach, addressing returns, including inventory, and extending the model's application to diverse logistics networks.

Keywords: *Omnichannel, BOPS, Capacity Sharing, Order Fulfillment, Vehicle Routing Problem*

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5.1 Summary

This study addresses a more generalized and realistic capacity-sharing vehicle routing problem with transfers in the context of BOPS retailing. This problem variant has been previously characterized by researchers such as Paul, Agatz, and Savelsbergh (2019) and Paul, Agatz, Spliet, and Koster (2019). In BOPS retailing, the orders are typically delivered at pick-up points (PUPs), often traditional retail stores. Major retailers like Walmart and Tesco have dedicated warehouses to fulfill online PUP orders. The operational setup involves two vehicles visiting the stores daily: one for replenishing store inventory and the other for delivering PUP orders. Joint planning of replenishment and PUP routes is a challenging task due to operational constraints, as replenishment routes are planned well in advance. In contrast, PUP routes are planned closer to the order fulfillment time due to the shorter lead times of PUP orders. However, the study explores the potential for leveraging the spare capacity of the replenishment vehicle by combining it with PUP orders, aiming to minimize the delivery distance for PUP orders.

The present study demonstrates significant savings in the delivery distance for PUP orders through capacity sharing. This reduces the delivery distance and brings tangible cost savings and intangible environmental benefits, such as reductions in greenhouse gas (GHG) emissions or carbon footprints. The key observations from the study are as follows:

- The total route distance for PUP order delivery with capacity sharing consistently exhibits lower values than the total route distance without capacity sharing, as illustrated in Table 3.1.
- When both the replenishment and PUP warehouses are co-located, capacity sharing results in greater distance savings compared to non-co-located warehouses, as outlined in Table 3.1.
- Higher distance savings are observed for scenarios with lower relative PUP demand than replenishment demand, as presented in Table 3.2.
- No clear trend is discernible in distance savings when considering the fraction of shared stores to the total number of retail stores, as indicated in Table 3.2.
- The increase in the probability of encountering capacitated retail stores corresponds to a decrease in distance savings, as demonstrated in Figure 3.1.
- An increase in the spare capacity of replenishment vehicles at the warehouse results in higher distance savings, as shown in Figure 3.2.
- Early initiation of PUP routes compared to replenishment routes leads to increased distance savings, as depicted in Figure 3.3.
- An increase in the fixed cost of transfer stores corresponds to decreased distance savings, as shown in Figure 3.4.
- The number of PUP vehicles required for capacity sharing is always equal to or higher than the number of PUP vehicles needed without capacity sharing, as detailed in Table 3.3.
- Distance savings are higher whenever the number of PUP routes without capacity

sharing is equal to the number of replenishment routes, as highlighted in Table 3.3.

- For co-located warehouses, the distance savings in joint planning are consistently greater than the distance savings in capacity sharing, as indicated in Table 3.4.
- For non-co-located warehouses, no superiority of joint planning is observed in terms of distance savings compared to capacity sharing, as presented in Table 3.4.

5.2 Managerial and Theoretical Contributions

5.2.1 Theoretical Contributions

The present work builds upon the previous studies conducted by Paul, Agatz, Spliet, and Koster (2019) and Paul, Agatz, and Savelsbergh (2019) and extends their findings by proposing a more generalized and realistic capacity-sharing vehicle routing problem for PUP order fulfillment. Unlike the previous studies, which focused on single replenishment and PUP routes, this study considers the scenario of multiple routes for both replenishment and PUP vehicles.

The main theoretical contribution lies in developing a MILP model and proposing efficient heuristics to address the capacity-sharing vehicle routing problem. The study provides a systematic and structured approach to address the complexities of the capacity-sharing vehicle routing problem.

The research offers a more accurate representation of real-world scenarios in BOPS retailing by considering multiple replenishment and PUP routes. The proposed model and its solutions have the potential to enhance the efficiency of order fulfillment processes by minimizing delivery distances and optimizing vehicle utilization. This, in turn, leads to tangible cost savings and intangible environmental benefits, such as reduced greenhouse gas emissions.

The theoretical contributions made in this study contribute to advancing the understanding of capacity-sharing vehicle routing for PUP order fulfillment.

5.2.2 Managerial Contributions

The present study has significant managerial implications, as it proposes a generalized distance-saving model for PUP order delivery. The main implications for managers are as follows:

- Optimal Route Planning: The model assists managers in finding the optimal route for PUP order delivery, considering distance minimization. By incorporating real-time data and operational constraints, managers can make informed decisions regarding route planning, leading to improved efficiency and reduced delivery costs.

- Transfer Location Optimization: The model helps managers identify the transfer locations where the demand of PUP orders can be efficiently transferred to the replenishment vehicle. By strategically selecting these transfer locations, managers can minimize the total distance and cost associated with PUP order delivery. This optimization contributes to overall cost savings and enhanced resource utilization.

- Efficient Store Selection: The model enables managers to identify and enlist the retail stores whose demand can be transferred at the designated transfer locations. Managers can streamline the store selection process by considering factors such as demand patterns, proximity to PUP warehouses, and vehicle capacities, resulting in improved order fulfillment efficiency.

- PUP Order Cost Reduction: By leveraging the insights provided by the model, managers can effectively reduce the overall cost of PUP order fulfillment. Optimized route planning, transfer location selection, and store enlistment contribute to cost savings through reduced transportation expenses, improved vehicle utilization, and minimized delivery distances.

These managerial implications highlight the practical significance of our the present research in the context of PUP order delivery. By utilizing the proposed distance-saving model, managers can make informed decisions and implement effective strategies to enhance the efficiency, cost-effectiveness, and overall performance of their PUP order fulfillment processes.

5.3 Limitations & Future Scope

The present thesis acknowledges certain limitations that should be considered. Although it proposes a mathematical model for the generalized distance-saving vehicle routing problem with store transfers for PUP order delivery, it has not yet validated the model through real-life industry implementation. However, the model has demonstrated significant savings on simulated data, providing confidence in its potential practical applicability.

One limitation of the model is that it assumes a homogeneous fleet for replenishment and PUP vehicles. In future studies, it would be beneficial to explore the use of a heterogeneous fleet, considering variations in vehicle capacities, capabilities, and characteristics, which may further enhance the optimization of the routing problem. Additionally, while the capacity sharing approach generally involves a higher number of PUP vehicles, the inclusion of the capital cost associated with using PUP vehicles would provide a more realistic analysis. [21] Furthermore, the loading and unloading time at the transfer location has not been considered in our model.

There is potential for the implementation of multi-objective goal programming to extend the optimization scope, encompassing factors such as cost, lead time, and space utilization. Given the non-uniform sizes of various product types, incorporating product density (mass/volume) for spare capacity consideration would significantly enhance the realism of the model.

Additionally, the model does not allow partial deliveries through transferring and direct

[21] We have observed that, when implementing capacity sharing, the total number of PUP vehicles needed equals or exceeds that required without capacity sharing (refer to section 3.8). Utilizing a greater number of vehicles incurs fixed capital costs related to drivers, acquisition, maintenance, insurance, etc. Considering these fixed costs would contribute to more realistic scenarios.

delivery. Allowing such partial deliveries could potentially increase the distance savings (refer to appendix E.1). Similarly, the model restricts multiple visits of PUP vehicles to retail stores, except for the replenishment warehouse. Allowing multiple PUP vehicle visits to retail stores may lead to additional savings (refer to appendix E.2). These aspects can be explored in future research to further refine the optimization of the routing problem.

Further avenues for future research include considering the replenishment route cost as endogenous and optimizing both replenishment and PUP routes simultaneously. Additionally, the model does not account for cases where certain retail stores exclusively serve as PUP stores. Including such pure PUP stores in the model would provide valuable insights into their impact on the overall routing problem. Furthermore, the consideration of returns associated with replenishment and PUP deliveries could be integrated, addressing the shared capacity vehicle routing problem with pickups and deliveries.

The current thesis does not explore the potential use of spare capacity in tomorrow's replenishment vehicle to deliver today's PUP orders. Future research could investigate compensatory strategies, such as discounts or less premium services, to optimize logistics operations in scenarios involving delayed PUP deliveries. Lastly, it is worth noting that the model is specific to the BOPS context. However, it can extend its application to other logistics networks where collaboration and capacity sharing are feasible. Exploring these possibilities would contribute a more comprehensive understanding of collaborative routing optimization in various logistical scenarios.

5.4 Way Forward

In conclusion, this thesis presents a comprehensive optimization model for minimizing delivery distance in BOPS order fulfillment, emphasizing piggybacking PUP orders on the replenishment vehicles. The imperative ahead is to enhance solution accessibility for both the Operations Research and Management Science (ORMS) community and industry.

To bridge the academia-industry gap, proposed initiatives include creating user-friendly interfaces and visualization tools tailored to practitioners with diverse ORMS expertise. Collaborative efforts with industry partners will offer practical insights, refining the model to align with industry needs. Additionally, conducting workshops, webinars, or training sessions will educate practitioners on the model's benefits and applications, promoting effective implementation in practical settings.

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Appendix A

A.1 Comparison between Paul, Agatz, and Savelsbergh (2019) and Our Work

A.1.1 Differences

Sr. No.	Paul, Agatz, and Savelsbergh (2019)	Our Work
1	They considered a single replenishment route.	We consider multiple replenishment routes.
2	They considered a single PUP route.	We consider multiple PUP routes.
3	They assumed that the capacity of the replenishment vehicle and PUP vehicle is high enough to meet the replenishment demand of all stores and PUP demand of the shared retail stores, respectively.	We consider a more realistic scenario by assuming limited capacity for both the replenishment vehicle and PUP vehicles.
4	They considered both warehouses to be on the edge of the retail stores.	We consider both warehouses to be placed in-between the group of retail stores, not on the edges.

5	They used the Hamiltonian path heuristic to obtain the shortest path. They used a greedy approach to find the total number of transfer stores.	We used a greedy approach to find the set of transfer and transferring stores.

Table A.1: Difference between Paul, Agatz, and Savelsbergh (2019) and our work

A.1.2 Similarities

Sr. No.	Paul, Agatz, and Savelsbergh (2019)	Our Work
1	They considered synchronization of the both replenishment and PUP vehicles.	We also consider synchronization of the both replenishment and PUP vehicles.
2	They considered two cases: (i) Both replenishment and PUP warehouses at same locations, and (ii) Both replenishment and PUP warehouses at different locations.	We also consider two cases: (i) Both replenishment and PUP warehouses at same locations, and (ii) Both replenishment and PUP warehouses at different locations.
3	They considered same starting time for the replenishment and PUP routes.	We also consider the same starting time for the replenishment and PUP routes.

Table A.2: Similarities between Paul, Agatz, and Savelsbergh (2019) and our work

A.2 Merit of our work over Paul, Agatz, Spliet, and Koster (2019) and Paul, Agatz, and Savelsbergh (2019)

The merit of our work compared to Paul, Agatz, Spliet, and Koster (2019) and Paul, Agatz, and Savelsbergh (2019) are summarized in Table A.3.

Criteria		Paul, Agatz, Spliet, and Koster (2019)	Paul, Agatz, and Savelsbergh (2019)	Present Work
No. of Replenishment Route	Single		✓	
	Multiple	✓		✓
No. of PUP Routes	Single		✓	
	Multiple	✓		✓
Replenishment Route Cost	Exogeneous	✓	✓	✓
	Endogeneous			
Transfer Point(s)	Single	✓		
	Multiple		✓	✓
Capacity of transfer point	Limited		✓	✓
	Unlimited	✓		
Transfer Cost at transfer points		✓	✓	✓
Capacity of Vehicles	Limited	✓		✓
	Unlimited		✓	
Fleet	Homogeneous	✓		✓
	Heterogeneous			
Replenishment Demand at stores	Constant		✓	
	Random	✓		✓
PUP Demand at stores	Constant		✓	
	Random	✓		✓
Type of Stores	Pure Replenishment	✓	✓	✓
	Pure PUP			
	Shared	✓	✓	✓
Type of Warehouses	Dedicated	✓	✓	✓
	Common			
Location of both dedicated Warehouse	Same Place		✓	✓
	Different Place	✓	✓	✓

Table A.3: Merit of our work over Paul, Agatz, Spliet, and Koster (2019) and Paul, Agatz, and Savelsbergh (2019)

Appendix B

B.1 Explanation of set $\overline{\mathcal{R}}$

The $\overline{\mathcal{R}}$ is a collection of sets of replenishment routes. It consists of the details of the replenishment routes, i.e., stores visited in each route, the sequence of store visits, arrival time of the replenishment vehicle at stores, and spare capacity of the replenishment vehicle while leaving the warehouse.

Let's assume that there are two replenishment routes, as mentioned in Figure B.1.

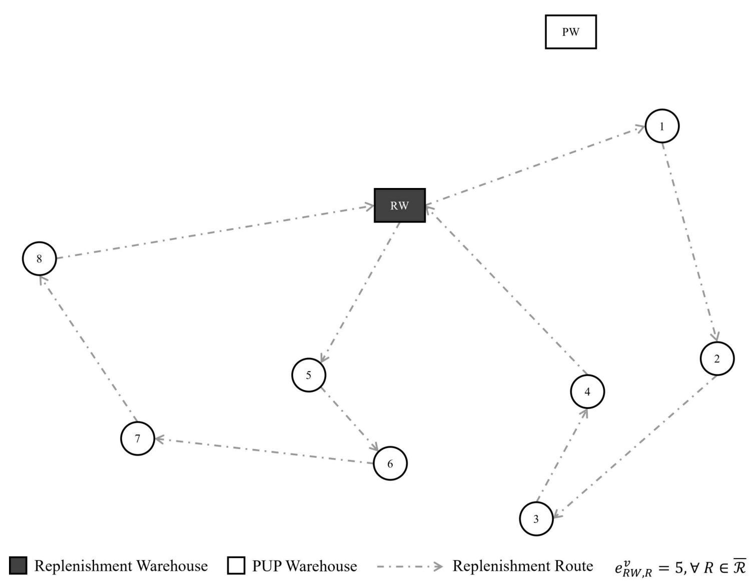

Figure B.1: Replenishment routes

The set $\overline{\mathcal{R}}$ can be written as,

$\overline{\mathcal{R}} = \{\{R_1:$ Route: [RW, S01, S02, S03, S04, RW],
Position: [RW: 0, S01: 1, S02: 2, S03: 3, S04: 4],
Arrival Time: [RW: 0, S01: 23.19, S02: 49.32, S03: 66.52, S04: 78.69],
Store Spare Capacity: [RW: 1000, S01: 10, S02: 10, S03: 10, S04: 10],
Vehicle Spare Capacity: [RW: 5]},

$\{R_2:$ Route: [RW, S05, S06, S07, S08, RW],
Position: [RW: 0, S05: 1, S06: 2, S07: 3, S08: 4],
Arrival Time: [RW: 0, S05: 22.80, S06: 33.98, S07: 55.57, S08: 83.03],
Store Spare Capacity: [RW: 1000, S05: 10, S06: 10, S07: 10, S08: 10],
Vehicle Spare Capacity: [RW: 5]}}.

B.2 Revised Formulation

The updated revised MILP formulation, which includes replenishment warehouse proxies, is mentioned below:

$$Z = min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{B.1}$$

PUP order fulfillment distance minimization objective function

Subject to,

Demand Constraints

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ij} + \sum_{\substack{i \in R \\ i < j}} z_{ij} = 1, \quad \forall R \in \overline{\mathcal{R}}, j \in (S \cap R) \tag{B.2}$$

Demand either directly delivered or transfer to replenishment route

Routing Constraints

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ij} = \sum_{\substack{i \in V \\ i \neq j}} x_{ji}, \quad \forall j \in N_R \tag{B.3}$$

Vehicles enter the store, it also leaves the store

$$z_{ij} \leq y_i, \quad \forall R \in \overline{\mathcal{R}}, i \in R, j \in (S \cap R), i < j \tag{B.4}$$

Transfer can only be possible at the transfer store

$$y_i \leq \sum_{\substack{j \in V \\ i \neq j}} x_{ji}, \quad \forall i \in N_R \tag{B.5}$$

PUP vehicle must visit the transfer store

Time Constraints

$$t_i \leq a_i + (1 - y_i)M_1, \quad \forall i \in N_R \tag{B.6}$$

PUP vehicle must visit transfer store before replenishment vehicle

$$t_i + c_{ij} \leq t_j + (1 - x_{ij})M_2, \quad \forall i \in V, j \in N_R \tag{B.7}$$

Update PUP vehicle time

Capacity Constraints

$$\sum_{\substack{j \in (S \cap R) \\ i < j}} z_{ij} q_j \leq w_i, \quad \forall R \in \overline{\mathcal{R}}, i \in (N \cap R) \tag{B.8}$$

Transferred demand must not exceed the actual transfer capacity - 1

$$\sum_{j \in S} z_{ij} q_j \leq w_i, \quad \forall i \in RW \tag{B.9}$$

Transferred demand must not exceed the actual transfer capacity - 2

$$w_i \leq e_i^s, \quad \forall i \in N_R \tag{B.10}$$

Actual transfer capacity must be less than store capacity

$$w_i \leq e_{RW,R}^v + \sum_{\substack{t \in (N \cap R) \\ t \leq i}} d_t - \sum_{\substack{t \in R \\ t < i}} \sum_{\substack{j \in (S \cap R) \\ i < j}} z_{tj} q_j, \quad \forall R \in \overline{\mathcal{R}}, i \in (N \cap R) \tag{B.11}$$

Actual transfer capacity must not exceed the vehicle capacity - 1

$$w_i \leq \sum_{R \in \overline{\mathcal{R}}} e_{RW,R}^v, \quad \forall i \in RW \tag{B.12}$$

Actual transfer capacity must not exceed the vehicle capacity - 2

$$\sum_{i \in RW} \sum_{j \in (S \cap R)} z_{ij} q_j \leq e_{RW,R}^v, \quad \forall R \in \overline{\mathcal{R}} \tag{B.13}$$

Actual transfer capacity must not exceed the vehicle capacity - 3

Subtour Interdependent Constraints

$$m_i = \sum_{\substack{j \in (S \cap R) \\ i<j}} z_{ij} q_j + \left(1 - \sum_{\substack{t \in R, i \in (S \cap R) \\ t<i}} z_{ti} \right) q_i, \quad \forall R \in \overline{\mathcal{R}}, i \in (N \cap R) \tag{B.14}$$

Unloading total PUP demand at the store - 1

$$m_i = \sum_{j \in S} z_{ij} q_j, \quad \forall i \in RW \tag{B.15}$$

Unloading total PUP demand at the store - 2

$$u_i + m_j \leq u_j + (1 - x_{ij}) Q_P, \quad \forall i \in N_R, j \in N_R \tag{B.16}$$

MTZ sub tour elimination constraint

Domain Constraints

$$m_i \leq u_i \leq Q_P, \quad \forall i \in N_R \tag{B.17}$$

Decision Variable domain - 1

$$x_{ij} \in \{0, 1\}, \quad \forall i \in V, j \in V \tag{B.18}$$

Decision Variable domain - 2

$$y_i \in \{0, 1\}, \quad \forall i \in N_R \tag{B.19}$$

Decision Variable domain - 3

$$0 \leq z_{ij} \leq 1, \quad \forall i \in N_R, j \in S \tag{B.20}$$

Decision Variable domain - 4

$$t_i, w_i, m_i, u_i \in \mathbb{R}_{\geq 0}, \quad \forall i \in N_R \tag{B.21}$$

Decision Variable domain - 5

Appendix C

C.1 Savings from the capacity sharing

In this appendix, we provide an analysis of the distance savings resulting from the implementation of capacity sharing between PUP routes and replenishment routes. The detailed representation of these distance savings can be found in Table C.1. To conduct this analysis, we generated instances with parameters $n = 12, 16, \alpha = 0.25, 0.5, 0.75, 1, \beta = 0.3, 0.5, 0.8$, and $\rho = 1$. This comprehensive evaluation allows us to assess the advantages derived from the utilization of shared capacities within the considered routing framework.

C.2 Effect of store transfer capacity

The data corresponding to Figure 3.1 is presented in Table C.2. This table illustrates the impact of the store transfer capacity on the achieved distance savings.

C.3 Effect of spare capacity of replenishment vehicle at replenishment warehouse

The data corresponding to Figure 3.2 is provided in Table C.3. This table showcases the influence of the spare capacity of the replenishment vehicle when leaving the replenishment warehouse on the achieved distance savings.

Instance Info			β	Non Co-located Warehouses							Co-located Warehouses						
				△ Transport Cost (%)	Stores Visited		Transfer Points		Stores Transferred		△ Transport Cost	Stores Visited		Transfer Points		Stores Transferred	
|N|	α	|S|			#	%	#	%	#	%		#	%	#	%	#	%
12	0.25	3	0.3	26.4	2.2	73.3	1.0	45.5	1.2	40.0	74.5	1.6	53.3	1.2	75.0	2.4	80.0
			0.5	16.8	2.8	93.3	1.0	35.7	1.0	33.3	78.1	1.6	53.3	1.2	75.0	2.4	80.0
			0.8	6.0	3.0	100.0	0.4	13.3	0.4	13.3	15.6	3.4	113.3	1.6	47.1	1.0	33.3
	0.50	6	0.3	27.4	4.2	70.0	1.8	42.9	2.6	43.3	59.3	3.2	53.3	2.4	75.0	4.4	73.3
			0.5	25.6	4.2	70.0	2.4	57.1	2.6	43.3	58.0	3.8	63.3	2.6	68.4	4.2	70.0
			0.8	4.5	6.0	100.0	0.6	10.0	0.6	10.0	31.1	6.2	103.3	5.0	80.6	2.8	46.7
	0.75	9	0.3	23.2	5.4	60.0	2.4	44.4	3.8	42.2	57.4	4.2	46.7	3.4	81.0	6.4	71.1
			0.5	17.1	6.0	66.7	3.2	53.3	3.4	37.8	51.1	5.0	55.6	3.6	72.0	5.6	62.2
			0.8	0.6	9.0	100.0	0.4	4.4	0.4	4.4	30.5	8.0	88.9	6.2	77.5	3.0	33.3
	1.00	12	0.3	19.9	8.2	68.3	3.0	36.6	4.4	36.7	59.9	4.4	36.7	3.8	86.4	8.6	71.7
			0.5	18.5	8.2	68.3	4.4	53.7	4.4	36.7	42.8	7.0	58.3	4.2	60.0	6.0	50.0
			0.8	0.0	12.0	100.0	0.0	0.0	0.0	0.0	29.2	10.0	83.3	7.4	74.0	3.0	25.0
16	0.25	4	0.3	15.3	3.6	90.0	1.2	33.3	1.6	40.0	65.4	2.2	55.0	1.4	63.6	3.2	80.0
			0.5	15.2	3.4	85.0	1.2	35.3	1.4	35.0	70.2	2.2	55.0	1.4	63.6	3.2	80.0
			0.8	6.1	4.2	105.0	1.0	23.8	1.0	25.0	28.7	4.8	120.0	3.6	75.0	2.4	60.0
	0.50	8	0.3	23.2	6.2	77.5	2.4	38.7	3.4	42.5	64.9	4.2	52.5	2.2	52.4	5.6	70.0
			0.5	19.4	6.0	75.0	2.6	43.3	3.2	40.0	63.9	4.2	52.5	2.4	57.1	5.6	70.0
			0.8	7.3	8.2	102.5	1.4	17.1	1.4	17.5	28.3	8.4	105.0	5.4	64.3	3.6	45.0
	0.75	12	0.3	27.0	8.0	66.7	2.8	35.0	5.2	43.3	59.7	5.6	46.7	3.6	64.3	7.8	65.0
			0.5	21.1	8.6	71.7	3.8	44.2	4.4	36.7	55.8	6.2	51.7	4.2	67.7	7.4	61.7
			0.8	4.2	12.2	101.7	3.2	26.2	0.8	6.7	35.6	10.2	85.0	8.4	82.4	4.4	36.7
	1.00	16	0.3	24.7	10.5	65.6	4.0	38.1	6.5	40.6	62.7	5.4	33.8	4.8	88.9	11.6	72.5
			0.5	18.2	11.4	71.3	5.2	45.6	5.2	32.5	47.2	9.0	56.3	5.0	55.6	8.0	50.0
			0.8	0.0	16.0	100.0	0.0	0.0	0.0	0.0	30.7	13.0	81.3	9.6	73.8	4.0	25.0

Table C.1: Distance savings from capacity sharing ($\rho = 1$)

Instance Info		ρ	Δ transport distance (%)	# of stores visited	# of transfer stores	# of stores transferred
|N|	|S|					
8	6	0	34.30	4.4	1.6	2.2
		0.25	21.43	5.0	1.4	1.6
		0.5	21.27	4.8	1.2	1.4
		0.75	16.19	6.0	2.4	1.2
		1	5.23	6.6	1.8	0.6
12	9	0	17.06	5.8	3.2	3.4
		0.25	11.29	7.4	1.8	1.8
		0.5	6.54	7.8	1.2	1.2
		0.75	0.66	8.6	0.4	0.4
		1	0.00	9.0	0.0	0.0

Table C.2: Effect of ρ on distance savings [non co-located warehouses] ($\alpha = 0.75, \beta = 0.5$)

Instance Info		$e^{v}_{RW,R}$	$\triangle$ transport distance (%)	# of stores visited	# of transfer stores	# of stores transferred
|N|	|S|					
8	6	0	35.20	4.2	2.2	2.2
		5	61.92	3.8	2.4	3.4
		10	82.88	2.2	2.0	4.8
		15	88.93	1.8	1.2	5.2
		20	100.00	1.0	1.0	6.0
12	9	0	28.32	6.2	4.0	4.0
		5	53.11	5.0	3.4	5.4
		10	74.39	3.0	2.2	7.0
		15	88.14	2.2	1.0	7.8
		20	100.00	1.0	1.0	9.0

Table C.3: Effect of $e^{v}_{RW,R}$ on distance savings [co-located warehouses] ($\alpha = 0.75, \beta = 0.5, \rho = 1$)

C.4 Effect of earliest start time of the PUP vehicle

The data corresponding to Figure 3.3 can be found in Table C.4. This table presents the impact of the earliest start time of the PUP vehicle on the distance savings achieved in the context of BOPS retailing.

C.5 Effect of transfer cost at store

The data corresponding to Figure 3.4 is provided in Table C.5. This table demonstrates the influence of the fixed cost associated with transferring at the transfer stores on the achieved distance savings.

Instance Info		δ	$\triangle$ transport distance (%)	# of stores visited	# of transfer stores	# of stores transferred
|N|	|S|					
8	6	0	34.30	4.4	1.6	2.2
		10	37.38	3.8	2.2	2.8
		20	38.08	3.6	2.4	3.0
		30	45.35	3.4	2.8	3.8
12	9	0	17.06	5.8	3.2	3.4
		10	24.50	5.4	3.6	4.0
		20	24.50	5.4	3.6	4.0
		30	38.84	5.4	4.0	5.2

Table C.4: Effect of earliest start time of the PUP vehicle on distance savings [non co-located] ($\alpha = 0.75, \beta = 0.5, \rho = 1$)

Instance Info						
N	S	η	Δ distance savings (%)	# of stores visited	# of transfer stores	# of stores transferred
12	9	0	36.30	4.2	3.0	5.4
		20	23.79	5.0	2.4	4.6
		40	12.97	5.8	2.0	3.8
		60	6.07	7.4	0.8	1.6
		80	3.00	7.8	0.6	1.2
		100	0.40	8.4	0.4	0.8
8	6	0	53.30	2.8	2.0	3.8
		20	40.03	2.8	2.0	3.8
		40	28.23	3.4	1.8	3.0
		60	19.45	4.4	1.0	2.0
		80	12.82	4.4	1.0	2.0
		100	7.56	4.6	0.8	1.6

Table C.5: Effect of store transfer cost on distance savings [co-located warehouse] ($\alpha = 0.75, \beta = 0.5, \rho = 0$)

Appendix D

D.1 Sample Problem

The accompanying figure D.1 illustrates a sample toy problem comprising 12 retail store

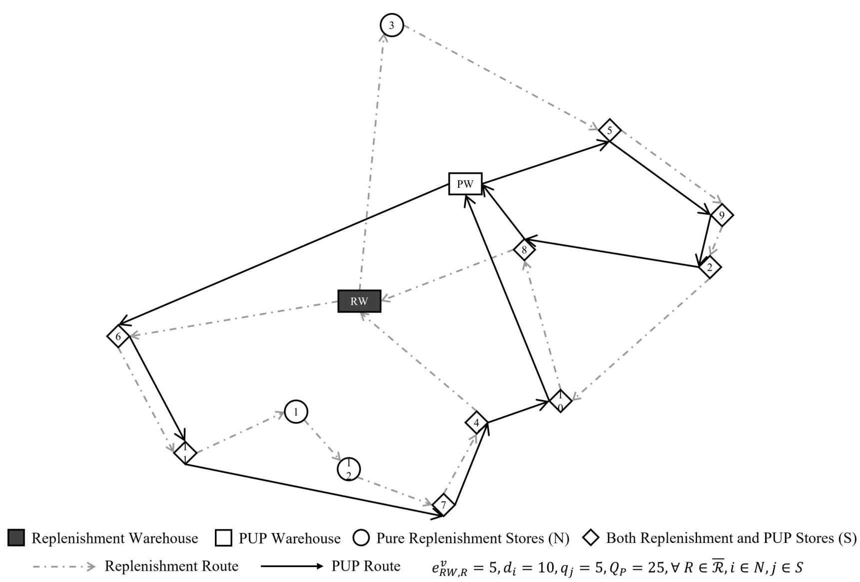

Figure D.1: Sample Toy Problem

The list of shared retail stores that exhibit both replenishment and PUP demand consists of the following: $[S05, S09, S02, S10, S08, S06, S11, S07, S04]$.

Table D.1 presents the notations utilized in the heuristics.

Notation	Description
N	Set of retail stores
RW	Replenishment warehouse
NR	Set of retail stores and replenishment warehouse, $N \cup RW$
S	Set of shared retail stores, $S \subseteq N$
d_i	Replenishment demand
q_i	PUP demand
a_i	Arrival time of replenishment vehicle at store i
t_i	Arrival time of PUP vehicle at store i
e_i^s	Store transfer capacity at store i
$e_{i,R}^v$	Vehicle transfer capacity at store i of replenishment route R, $\forall R \in \overline{\mathcal{R}}$
T_i^s	Aggregate PUP demand that can transferred at store i
M_i	$min(e_i^s, e_{i,R}^v, T_i^s)$
R_i^1	$\frac{M_i}{c_{oi}}$
R_i^2	$\frac{c_{PWi}}{q_i}$

Table D.1: Notations used in the Heuristics

Table D.2 displays the distances between various retail stores and warehouses.

Table D.3 presents the remaining parametric values.

Store	q_i	di	Position	a_i	e_i^s	$e_{i,R}^v$
RW	0	0	0	0.00	1000	5
S03	0	10	1	54.15	10	15
S05	5	10	2	96.60	10	25
S09	5	10	3	115.81	10	35
S02	5	10	4	125.86	10	45
S10	5	10	5	152.88	10	55
S08	5	10	6	179.19	10	65

S06	5	10	1	37.22	10	15
S11	5	10	2	64.68	10	25
S01	0	10	3	81.44	10	35
S12	0	10	4	92.62	10	45
S07	5	10	5	111.30	10	55
S04	5	10	6	123.47	10	65

Table D.3: Parametric Values

In the current experiments, we assume that both the PUP and replenishment vehicles depart from the warehouse at time zero. The capacity of the PUP vehicle is denoted as $Q_P(=25)$.

D.2 Without Vehicle Capacity Sharing

In the absence of vehicle capacity sharing, the routes for the PUP vehicle are as follows: $[(PW \rightarrow S06 \rightarrow S11 \rightarrow S07 \rightarrow S04 \rightarrow S10 \rightarrow PW), (PW \rightarrow S05 \rightarrow S09 \rightarrow S02 \rightarrow S08 \rightarrow PW)]$. The total distance covered by these routes is 275.01 units. Figure D.2(a) illustrates the PUP route for the case without capacity sharing.

D.3 With Vehicle Capacity Sharing

In this section, we have examined the advantages gained from capacity sharing in BOPS retailing and have compared the optimal outcomes with heuristic A and heuristic B.

D.3.1 Optimum Solution

The PUP routes can be defined as $[(PW \rightarrow S09 \rightarrow S05 \rightarrow PW), (PW \rightarrow S11 \rightarrow S06 \rightarrow PW)]$, with a total distance covered of 212.62 units. The optimal PUP routes incorporating capacity sharing are illustrated in Figure D.2(b).

$c_{i,j}$	PW	RW	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10	S11	S12
PW	0	28.28	49.4	32.56	37.58	45.01	13.34	61.85	57.08	17.26	27.73	43.57	63.03	56.08
RW	28.28	0	22.8	44.05	54.15	31.4	40.22	37.22	40.72	23.19	45.71	35.47	34.83	32.02
S01	49.4	22.8	0	53.85	76.66	25.18	59.55	35.85	27.46	38.29	59.17	33.02	16.76	11.18
S02	32.56	44.05	53.85	0	68.82	33.97	27.31	81.02	44.2	21.59	10.05	27.02	70.58	54.08
S03	37.58	54.15	76.66	68.82	0	80.41	42.45	71.03	91.92	54.42	61.62	80.36	85.07	86.15
S04	45.01	31.4	25.18	33.97	80.41	0	50	59.81	12.17	28.28	42.01	8.25	41.05	21.19
S05	13.34	40.22	59.55	27.31	42.45	50	0	75.03	62.1	22.36	19.21	46.39	74.33	64.66
S06	61.85	37.22	35.85	81.02	71.03	59.81	75.03	0	63.29	60.41	82.87	66.76	27.46	45.61
S07	57.08	40.72	27.46	44.2	91.92	12.17	62.1	63.29	0	40.45	53	17.2	40.26	18.68
S08	17.26	23.19	38.29	21.59	54.42	28.28	22.36	60.41	40.45	0	22.56	26.31	54.08	42.44
S09	27.73	45.71	59.17	10.05	61.62	42.01	19.21	82.87	53	22.56	0	35.85	75.59	60.96
S10	43.57	35.47	33.02	27.02	80.36	8.25	46.39	66.76	17.2	26.31	35.85	0	49.16	29.41
S11	63.03	34.83	16.76	70.58	85.07	41.05	74.33	27.46	40.26	54.08	75.59	49.16	0	21.59
S12	56.08	32.02	11.18	54.08	86.15	21.19	64.66	45.61	18.68	42.44	60.96	29.41	21.59	0

Table D.2: Distance Matrix

Transfer stores	S09	S05	S05	S11	S11
transferred stores	S08	S02	S10	S04	S07
Fraction of demand transferred	1	1	1	1	1
Amount of demand transferred	5	5	5	5	5

Table D.4: Optimum Transfer set

D.3.2 Heuristic A

STEP 1: *Identify minimum number of vehicles required to meet the PUP demand*

The calculation for determining the minimum number of PUP vehicles needed to meet the PUP demand is expressed as follows: $\left\lceil \frac{\sum_{j \in S} q_j}{Q_P} \right\rceil = \left\lceil \frac{45}{25} \right\rceil = 2.$

STEP 2: *For Vehicle 1*

STEP 2a: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

Considering that both vehicles start at time zero, it is feasible to transfer the PUP demand to the replenishment route at the stores where the PUP vehicle can visit prior to the replenishment vehicle. The potential transfer locations encompass $[S03, S05, S09, S02, S10, S08, S11, S01, S12, S07, S04]$.

STEP 2b: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

To select an appropriate transfer store, it is recommended to consider two factors: (a) proximity to the current position of the PUP vehicle, and (b) the ability to transfer the maximum demand to the replenishment route. Therefore, the transfer store is chosen based on the highest ratio R_i^1, as shown in Table D.5. In the event of a tie in the ratio R_i^1, preference is given to the store that is closer to the location of the PUP vehicle.

i	c_{oi}	e_i^s	$e_{i,R}^v$	T_i^s	$M_i = min(e_i^s, e_{i,R}^v, T_i^s)$	$R_i^1 = \frac{M_i}{c_{oi}}$	
S03	37.58	10	15	25	10	0.266	
S05	13.34	10	25	20	10	0.750	←
S09	27.23	10	35	15	10	0.367	
S02	32.56	10	45	10	10	0.307	
S10	43.57	10	55	5	5	0.115	
S08	17.26	10	65	0	0	0.000	
S11	63.03	10	25	10	10	0.159	
S01	49.4	10	35	10	10	0.202	
S12	56.08	10	45	10	10	0.178	
S07	57.08	10	55	5	5	0.088	
S04	45.01	10	65	0	0	0.000	

Table D.5: Identifying transfer store (Heuristic A, Vehicle 1, Iteration I)

As R_i^1 is the highest for $S05$, it is chosen as the transfer location. Consequently, the updated location of the PUP vehicle is now $S05$. The revised PUP route becomes $[PW \rightarrow S05]$, covering a total distance of 13.34 units.

The demand of store $S05$ is fulfilled by the PUP vehicle, resulting in an accumulated delivery of 5 units by PUP vehicle 1.

STEP 2c: *Identifying transferred stores whose demand can be transferred at the transfer store*

To optimize the demand transfer to the replenishment route, it is advantageous to consider two factors: (a) distance from the PUP warehouse and (b) lower demand (indicating a higher potential for transferring more stores). The demand of stores with the highest ratio R_i^2 is prioritized for transfer. In the event of a tie in the ratio R_i^2, preference is given to the store that is farthest from the PUP warehouse.

Store	c_{PWi}	q_i	$R_i^2 = \frac{M_i}{c_{PWi}}$

S10	43.57	5	8.714	← (1)
S08	17.26	5	3.452	
S02	32.56	5	6.512	← (2)
S09	27.73	5	5.546	

Table D.6: Identifying transferred store (Heuristic A, Vehicle 1, Iteration I)

To ensure compliance with the maximum allowable demand at store $S05$ (denoted as $M_{S05} = 10$), the total transferred demand must not exceed this limit. Among all stores, the highest ratio R^2_{S10} suggests that we can transfer the maximum feasible demand of 5 units of store $S10$ to store $S05$. Consequently, the updated value of M_{S05} becomes 5.

Since the updated M_{S05} is still greater than zero, we proceed to consider the store with the second-highest ratio. store $S02$ possesses the highest ratio among the remaining stores, allowing us to transfer the maximum feasible demand from store $S02$ to store $S05$. We continue this process until M_{S05} reaches zero.

The transfer set is presented in Table D.7.

Transfer store	S05	S05
Transferred store	S10	S02
Fraction of demand transferred	1	1
Demand transferred	5	5

Table D.7: Transfer set (Heuristic A, Iteration I)

When a vehicle visits store $S05$, the total aggregate demand, whether directly delivered or transferred, amounts to 15. It is crucial to ensure that the aggregate demand transferred or delivered by any PUP vehicle remains below the vehicle capacity, which is set at 25.

The process of steps 2a, 2b, and 2c is iteratively repeated until the demand of all PUP stores is successfully transferred or until there are no feasible transfer locations available. In cases where the PUP demand of a retail store cannot be transferred due to certain constraints, the demand of that particular store is fulfilled directly by the PUP vehicle.

STEP 2d: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of possible transfer locations includes $[S09, S02, S10, S08, S01, S12, S07, S04]$.

STEP 2e: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

The values of $e^v_{i,R}$, e^s_i, and T^s_i are updated based on previous iteration(s).

i	c_{oi}	e^s_i	$e^v_{i,R}$	T^s_i	$M_i = min(e^s_i, e^v_{i,R}, T^s_i)$	$R^1_i = \frac{M_i}{c_{oi}}$	
S09	19.21	10	25	5	5	0.260	←
S02	27.31	10	40	5	5	0.183	
S10	46.39	10	55	5	5	0.108	
S08	22.36	10	65	0	0	0.000	
S01	59.55	10	35	10	10	0.168	
S12	64.66	10	45	10	10	0.155	
S07	62.1	10	55	5	5	0.081	
S04	50	10	65	0	0	0.000	

Table D.8: Identifying transfer store (Heuristic A, Vehicle 1, Iteration II)

As the highest R^1_i is associated with $S09$, it is chosen as the transfer location. Consequently, the updated location of the PUP vehicle becomes $S09$. The revised PUP route now consists of $[PW \rightarrow S05 \rightarrow S09]$, covering a total distance of 32.55 units.

The demand of store $S09$ is successfully delivered by the PUP vehicle, resulting in an accumulated delivery of 20 units by PUP vehicle 1.

STEP 2f: *Identifying transferred stores whose demand can be transferred at the transfer store*

Store	c_{PWi}	q_i	$R_i^2 = \frac{M_i}{c_{PWi}}$	
S08	17.26	5	3.452	←

Table D.9: Identifying transferred store (Heuristic A, Vehicle 1, Iteration II)

Transfer store	S05	S05	S09
Transferred store	S10	S02	S08
Fraction of demand transferred	1	1	1
Demand transferred	5	5	5

Table D.10: Transfer set (Heuristic A, Vehicle 1, Iteration II)

The PUP vehicle 1 has delivered an aggregate demand of 25 units, which is equivalent to its maximum vehicle capacity.

The final route of vehicle 1 is $[PW \rightarrow S05 \rightarrow S09 \rightarrow PW]$, encompassing a total distance of 60.28 units.

STEP 3: *For Vehicle 2*

STEP 3a: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of potential transfer locations includes [S03, S02, S10, S08, S11, S01, S12, S07, S04].

STEP 3b: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

i	c_{oi}	e_i^s	$e_{i,R}^v$	T_i^s	$M_i = min(e_i^s, e_{i,R}^v, T_i^s)$	$R_i^1 = \frac{M_i}{c_{oi}}$
S03	37.58	10	15	0	0	0.000
S02	32.56	10	35	0	0	0.000
S10	43.57	10	50	0	0	0.000

S08	17.26	10	65	0	0	0.000	
S11	63.03	10	25	10	10	0.159	
S01	49.4	10	35	10	10	0.202	←
S12	56.08	10	45	10	10	0.178	
S07	57.08	10	55	5	5	0.088	
S04	45.01	10	65	0	0	0.000	

Table D.11: Identifying transfer store (Heuristic A, Vehicle 2, Iteration I)

As R_i^1 is the highest for $S01$, it is chosen as the transfer location. Consequently, the updated location of the PUP vehicle becomes $S01$. The revised PUP route is now $[PW \rightarrow S01]$, covering a total distance of 49.4 units.

There is no PUP demand at store $S01$, resulting in an accumulated delivery of 0 units by PUP vehicle 2.

STEP 3c: *Identifying transferred stores whose demand can be transferred at the transfer store*

Store	c_{PWi}	q_i	$R_i^2 = \frac{M_i}{c_{PWi}}$	
S04	45.01	5	9.002	← (2)
S07	57.08	5	11.416	← (1)

Table D.12: Identifying transferred store (Heuristic A, Vehicle 2, Iteration I)

To ensure compliance with the maximum allowable demand at store $S01$ (denoted as $M_{S01} = 10$), the total transferred demand must not exceed this limit. Among all stores, the highest ratio R_{S07}^2 suggests that we can transfer the maximum feasible demand of 5 units from store $S07$ to store $S01$. Consequently, the updated value of M_{S01} becomes 5.

Since the updated M_{S01} is still greater than zero, we proceed to consider the store with the second-highest ratio. store $S04$ possesses the highest ratio among the remaining stores, allowing us to transfer the maximum feasible demand from store $S04$ to store $S01$. Now that the updated M_{S01} is zero, there are no further possible transfers at store $S01$.

The transfer set is presented in Table D.13.

Transfer store	S05	S05	S09	S01	S01
Transferred store	S10	S02	S08	S07	S04
Fraction of demand transferred	1	1	1	1	1
Demand transferred	5	5	5	5	5

Table D.13: Transfer set (Heuristic A, Iteration II)

When a vehicle visits store $S01$, the total aggregate demand delivered or transferred amounts to 10 units.

STEP 3d: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of possible transfer locations includes $[S02, S10, S08, S12, S07, S04]$.

STEP 3e: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

The values of $e^v_{i,R}$, e^s_i, and T^s_i are updated based on previous iteration(s).

i	c_{oi}	e^s_i	$e^v_{i,R}$	T^s_i	$M_i = min(e^s_i, e^v_{i,R}, T^s_i)$	$R^1_i = \frac{M_i}{c_{oi}}$
S02	53.85	10	35	0	0	0.000
S10	33.02	10	50	0	0	0.000
S08	38.29	10	65	0	0	0.000
S12	11.18	10	35	0	0	0.000
S07	27.46	10	50	0	0	0.000
S04	25.18	10	65	0	0	0.000

Table D.14: Identifying transfer store (Heuristic A, Vehicle 2, Iteration II)

Since all R^1_i values are zero, there are no remaining transfer stores available for vehicle

2. In such cases, we examine any retail stores with non-zero PUP demand and deliver the goods directly through the PUP vehicle.

STEP 3f: *Direct delivery of remaining stores with non-zero PUP demand*

In this step, since no transfer locations are available, we proceed with the direct delivery of PUP demand through the PUP vehicle.

First, we identify the PUP stores with non-zero demand and observe that the demand of retail stores $S06$ and $S11$ remains unmet.

Given that the current position of PUP vehicle 2 is at store $S01$, we select the retail store that is closest in distance to the current position of the PUP vehicle.

i	c_{oi}	
S11	16.76	←
S06	35.85	

Table D.15: Direct delivery (Heuristic A, Vehicle 2, Iteration I)

The next serving store, based on the current position of vehicle 2, is determined as $S11$. Consequently, the updated PUP route for vehicle 2 becomes $[PW \rightarrow S01 \rightarrow S11]$, resulting in a total distance covered of 66.52 units. Additionally, the accumulated demand delivered or transferred by vehicle 2 amounts to 15 units.

i	c_{oi}	
S06	27.46	←

Table D.16: Direct delivery (Heuristic A, Vehicle 2, Iteration II)

Based on the current position of vehicle 2, the next serving store is determined to be $S11$. Consequently, the updated PUP route for vehicle 2 becomes $[PW \rightarrow S01 \rightarrow S11 \rightarrow S06]$, resulting in a total distance covered of 93.98 units. The accumulated demand delivered or transferred by vehicle 2 is 20 units.

Since there are no PUP stores with non-zero demands remaining, vehicle 2 returns to the PUP warehouse. The final PUP route for vehicle 2 is $[PW \rightarrow S01 \rightarrow S11 \rightarrow S06 \rightarrow PW]$, leading to a total distance covered of 155.82 units.

Heuristic A Solution:

The PUP routes obtained through the application of Heuristic A are as follows: $(PW \rightarrow S09 \rightarrow S05 \rightarrow PW)$ for vehicle 1, and $(PW \rightarrow S01 \rightarrow S11 \rightarrow S06 \rightarrow PW)$ for vehicle 2. These routes result in a combined total distance covered of 291.84 units.

The calculated optimality gap for Heuristic A is 1.47%. This indicates that the solution obtained through Heuristic A is within 1.47% of the optimal solution. Figure D.3(a) illustrates the PUP routes considering capacity sharing under Heuristic A.

Transfer store	S05	S05	S09	S01	S01
Transferred store	S10	S02	S08	S07	S04
Fraction of demand transferred	1	1	1	1	1
Demand transferred	5	5	5	5	5

Table D.17: Heuristic A Transfer set

D.3.3 Heuristic B

STEP 1: *Identify minimum number of vehicles required to meet the PUP demand*

The calculation to determine the minimum number of PUP vehicles required to fulfill the PUP demand is expressed as follows: $\left\lceil \frac{\sum_{j \in S} q_j}{Q_P} \right\rceil = 2$

STEP 2: *For Vehicle 1*

STEP 2a: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The list of potential transfer locations includes the following stores: $[S03, S05, S09, S02, S10, S08, S11, S01, S12, S07, S04]$.

STEP 2b: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

i	c_{oi}	e_i^s	$e_{i,R}^v$	T_i^s	$M_i = min(e_i^s, e_{i,R}^v, T_i^s)$	q_i	$M_i + q_i$	$R_i^1 = \frac{M_i+q_i}{c_{oi}}$	
S03	37.58	10	15	25	10	0	10	0.266	
S05	13.34	10	25	20	10	5	15	1.124	←
S09	27.23	10	35	15	10	5	15	0.551	
S02	32.56	10	45	10	10	5	15	0.461	
S10	43.57	10	55	5	5	5	10	0.230	
S08	17.26	10	65	0	0	5	5	0.290	
S11	63.03	10	25	10	10	5	15	0.238	
S01	49.4	10	35	10	10	0	10	0.202	
S12	56.08	10	45	10	10	0	10	0.178	
S07	57.08	10	55	5	5	5	10	0.175	
S04	45.01	10	65	0	0	5	5	0.111	

Table D.18: Identifying transfer store (Heuristic B, Vehicle 1, Iteration I)

Considering that R_i^1 achieves its highest value at $S05$, this site is chosen as the transfer location. Consequently, the updated position of the PUP vehicle is at $S05$. The PUP route is defined as $[PW \rightarrow S05]$, covering a total distance of 13.34 units. Moreover, the PUP vehicle delivers the demand for store $S05$, resulting in an accumulated demand of 5 units fulfilled by PUP vehicle 1.

STEP 2c: *Identifying transferred stores whose demand can be transferred at the transfer store*

Store	c_{PWi}	q_i	$R_i^2 = \frac{M_i}{c_{PWi}}$	
S10	43.57	5	8.714	← (1)
S08	17.26	5	3.452	
S02	32.56	5	6.512	← (2)
S09	27.73	5	5.546	

Table D.19: Identifying transferred store (Heuristic B, Vehicle 1, Iteration I)

The total demand transferred at store $S05$ must not exceed the maximum capacity M_{S05}, which is set at 10. Among all the ratios, R^2_{S10} exhibits the highest value. Thus, we can transfer the maximum demand possible from store $S10$ to store $S05$, amounting to 5 units. Consequently, the value of M_{S05} is updated to 5.

Since the updated value of M_{S05} is still greater than zero, we proceed to identify the store with the second-highest ratio. Among the remaining stores, the ratio at store $S02$ is the highest. Hence, we transfer the maximum feasible demand from store $S02$ to store $S05$. This iterative process continues until M_{S05} reaches zero.

The transfer set is presented in Table D.20.

Transfer store	S05	S05
Transferred store	S10	S02
Fraction of demand transferred	1	1
Demand transferred	5	5

Table D.20: Transfer set (Heuristic B, Iteration I)

The total demand transferred upon the vehicle's visit to store $S05$ amounts to 15 units. It is important to note that the cumulative demand transferred or delivered by any PUP vehicle must not exceed the vehicle's capacity, which is set at 25 units.

STEP 2d: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of potential transfer locations consists of the following sites: $[S09, S02, S10, S08, S01, S12, S07, S04]$.

STEP 2e: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

The values of $e^{v}_{i,R}$, e^{s}_{i}, and T^{s}_{i} are updated using the information from the previous iteration(s).

i	c_{oi}	e^s_i	$e^v_{i,R}$	T^s_i	$M_i = min(e^s_i, e^v_{i,R}, T^s_i)$	q_i	$M_i + q_i$	$R^1_i = \frac{M_i+q_i}{c_{oi}}$	
S09	19.21	10	25	5	5	5	10	0.521	←
S02	27.31	10	40	5	5	0	5	0.183	
S10	46.39	10	55	5	5	0	5	0.108	
S08	22.36	10	65	0	0	5	5	0.224	
S01	59.55	10	35	10	10	0	10	0.168	
S12	64.66	10	45	10	10	0	10	0.155	
S07	62.1	10	55	5	5	5	10	0.161	
S04	50	10	65	0	0	5	5	0.100	

Table D.21: Identifying transfer store (Heuristic B, Vehicle 1, Iteration II)

Considering that R^1_i achieves its highest value at $S09$, this site is chosen as the transfer location. Consequently, the updated position of the PUP vehicle is at $S09$. The PUP route is defined as $[PW \rightarrow S05 \rightarrow S09]$, covering a total distance of 32.55 units. Moreover, the PUP vehicle delivers the demand for store $S09$, resulting in an accumulated demand of 20 units fulfilled by PUP vehicle 1.

STEP 2f: *Identifying transferred stores whose demand can be transferred at the transfer store*

Store	c_{PWi}	q_i	$R^2_i = \frac{M_i}{c_{PWi}}$	
S08	17.26	5	3.452	←

Table D.22: Identifying transferred store (Heuristic B, Vehicle 1, Iteration II)

Transfer store	S05	S05	S09
Transferred store	S10	S02	S08
Fraction of demand transferred	1	1	1
Demand transferred	5	5	5

Table D.23: Transfer set (Heuristic B, Vehicle 1, Iteration II)

The total demand delivered by PUP vehicle 1 amounts to 25 units, which matches the vehicle's capacity.

The final route for vehicle 1 is $[PW \rightarrow S05 \rightarrow S09 \rightarrow PW]$, covering a total distance of 60.28 units.

STEP 3: *For Vehicle 2*

STEP 3a: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of potential transfer locations includes the following sites: $[S03, S02, S10, S08, S11, S01, S12, S07, S04]$.

STEP 3b: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

i	c_{oi}	e_i^s	$e_{i,R}^v$	T_i^s	$M_i = min(e_i^s, e_{i,R}^v, T_i^s)$	q_i	$M_i + q_i$	$R_i^2 = \frac{M_i+q_i}{c_{oi}}$	
S03	37.58	10	15	0	0	0	0	0.000	
S02	32.56	10	35	0	0	0	0	0.000	
S10	43.57	10	50	0	0	0	0	0.000	
S08	17.26	10	65	0	0	0	0	0.000	
S11	63.03	10	25	10	10	5	15	0.238	←
S01	49.4	10	35	10	10	0	10	0.202	
S12	56.08	10	45	10	10	0	10	0.178	
S07	57.08	10	55	5	5	5	10	0.175	
S04	45.01	10	65	0	0	5	5	0.111	

Table D.24: Identifying transfer store (Heuristic B, Vehicle 2, Iteration I)

Given that R_i^1 attains the highest value for $S11$, this store is chosen as the transfer location. Consequently, the PUP vehicle's updated position is at $S11$. The PUP route is defined as $[PW \rightarrow S11]$, covering a total distance of 63.03 units.

Moreover, PUP vehicle 2 delivers a demand amounting to 5 units, resulting in an accumulated demand fulfilled by PUP vehicle 2.

STEP 3c: *Identifying transferred stores whose demand can be transferred at the transfer store*

Store	c_{PWi}	q_i	$R_i^2 = \frac{M_i}{c_{PWi}}$	
S04	45.01	5	9.002	← (2)
S07	57.08	5	11.416	← (1)

Table D.25: Identifying transferred store (Heuristic B, Vehicle 2, Iteration I)

The total demand transferred at store $S11$ must not exceed the maximum capacity M_{S11}, which is set at 10. Among all the ratios, R^2_{S07} exhibits the highest value. Thus, we can transfer the maximum possible demand from store $S07$ to store $S11$, amounting to 5 units. Consequently, the value of M_{S11} is updated to 5.

Since the updated value of M_{S11} is still greater than zero, we proceed to identify the store with the second-highest ratio. Among the remaining stores, the ratio at store $S04$ is the highest. Hence, we transfer the maximum feasible demand from store $S04$ to store $S11$. Now that M_{S11} is updated to zero, no further transfers are possible at store $S11$.

The transfer set is presented in Table D.26.

Transfer store	S05	S05	S09	S11	S11
Transferred store	S10	S02	S08	S07	S04
Fraction of demand transferred	1	1	1	1	1
Demand transferred	5	5	5	5	5

Table D.26: Transfer set (Heuristic B, Iteration II)

The total demand transferred when a vehicle visits store $S11$ amounts to 15 units.

STEP 3d: *Identify set of stores where PUP vehicle can reach before replenishment vehicle*

The set of potential transfer locations includes the following stores: $[S02, S10, S08, S12, S07, S04]$.

STEP 3e: *Identify the transfer store where PUP demand can be transferred to the replenishment route*

The value of the $e^v_{i,R}$, e^s_i and T^s_i are updated based on previous iteration(s).

i	c_{oi}	e^s_i	$e^v_{i,R}$	T^s_i	$M_i = min(e^s_i, e^v_{i,R}, T^s_i)$	q_i	$M_i + q_i$	$R^2_i = \frac{M_i+q_i}{c_{oi}}$
S02	53.85	10	35	0	0	0	0	0.000
S10	33.02	10	50	0	0	0	0	0.000
S08	38.29	10	65	0	0	0	0	0.000
S12	11.18	10	35	0	0	0	0	0.000
S07	27.46	10	50	0	0	0	0	0.000
S04	25.18	10	65	0	0	0	0	0.000

Table D.27: Identifying transfer store (Heuristic B, Vehicle 2, Iteration II)

Since all R^1_i values are zero, there are no remaining transfer stores for vehicle 2. In this case, we proceed to examine any retail stores with non-zero PUP demand and directly deliver the required items using the PUP vehicle.

STEP 3f: *Direct delivery of remaining stores with non-zero PUP demand*

Considering the absence of available transfer locations, in the current step, we proceed with directly delivering the PUP demand using the PUP vehicle.

Firstly, we identify the PUP stores with non-zero demand, and we observe that the demand of retail store $S06$ remains unfulfilled.

Since the current position of PUP vehicle 2 is store $S11$, we select the retail store that has the minimum distance from the current position of the PUP vehicle.

i	c_{oi}	
S06	27.46	←

Table D.28: Direct delivery (Heuristic B, Vehicle 2, Iteration I)

The subsequent store to be served is determined as $S06$, resulting in the updated PUP route for vehicle 2 as $[PW \rightarrow S11 \rightarrow S06]$. The total distance covered by this route amounts to 90.49 units.

In the scenario where there are no PUP stores with non-zero demands remaining, vehicle 2 returns to the PUP warehouse. The final PUP route for vehicle 2 is defined as $[PW \rightarrow S11 \rightarrow S06 \rightarrow PW]$, covering a total distance of 152.34 units.

Heuristic B Solution:

The PUP routes are given as $[(PW \rightarrow S09 \rightarrow S05 \rightarrow PW), (PW \rightarrow S11 \rightarrow S06 \rightarrow PW)]$, covering a total distance of 212.62 units. The optimality gap is 0%. The PUP routes with capacity sharing are illustrated in Figure D.3(b).

Transfer store	S05	S05	S09	S11	S11
Transferred store	S10	S02	S08	S07	S04
Fraction of demand transferred	1	1	1	1	1
Demand transferred	5	5	5	5	5

Table D.29: Heuristic B Transfer set

D.4 Comparison of results

In the present section, we conducted a graphical comparison of the distances traveled by the PUP vehicle to fulfill the PUP demands, considering the optimal solution, heuristic A, and heuristic B.

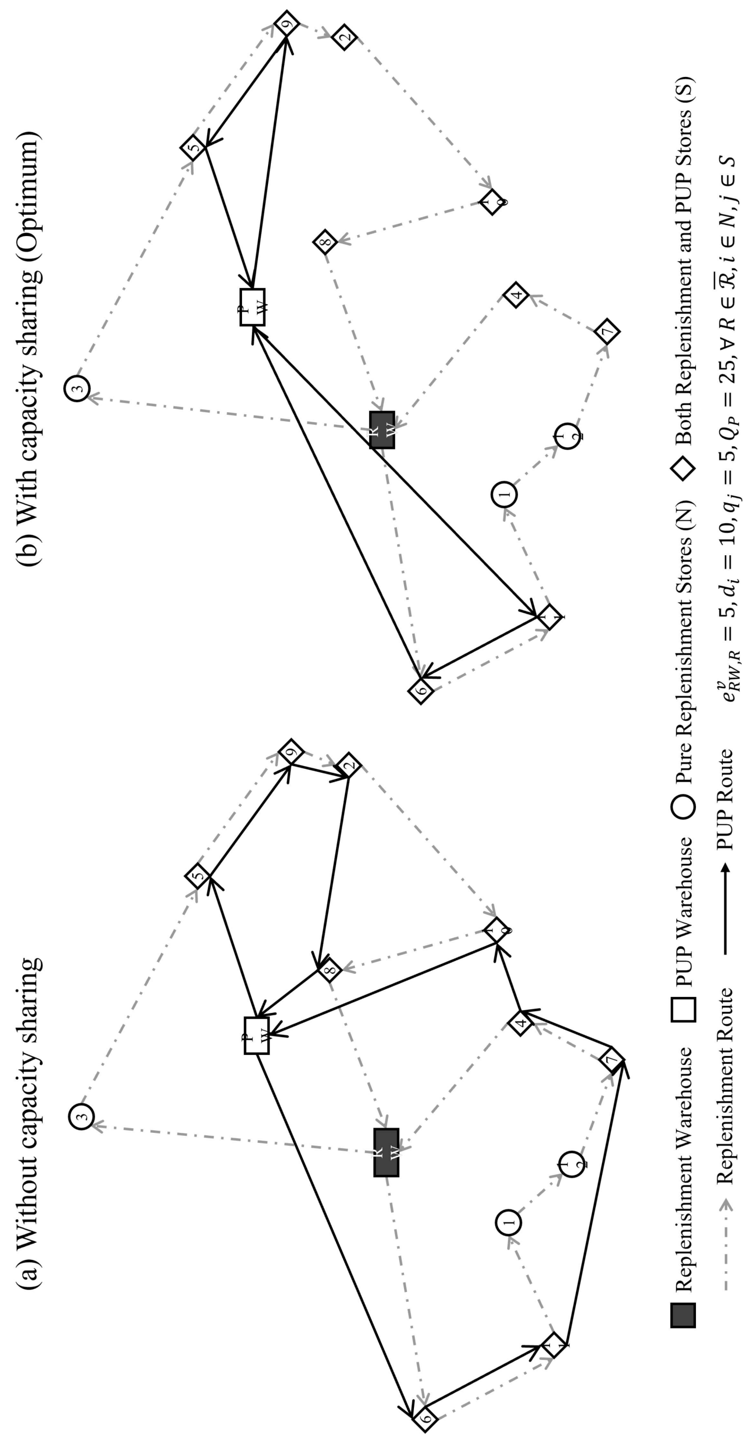

Figure D.2: Comparison between with and without capacity sharing

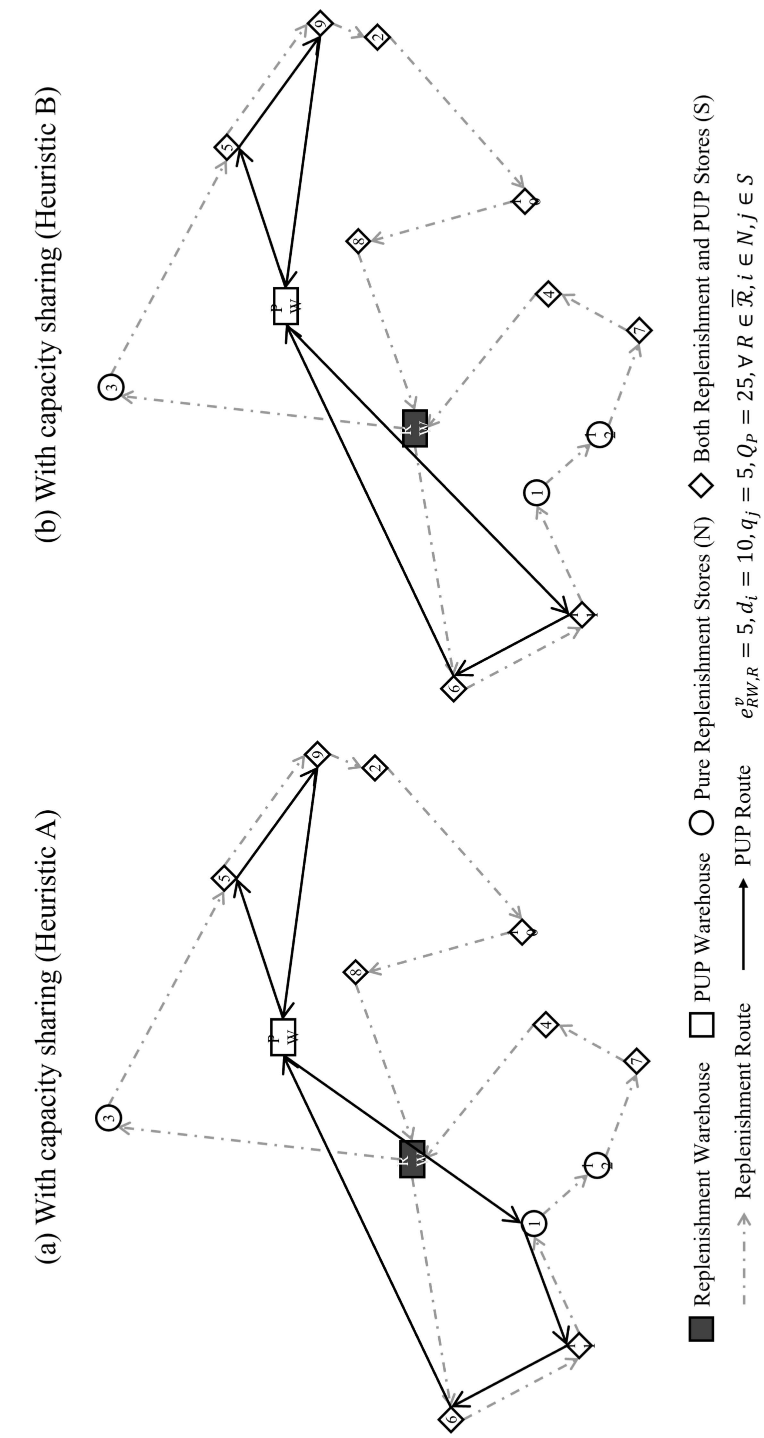

Figure D.3: Comparison between Heuristic A and Heuristic B in capacity sharing case

Appendix E

In this appendix, we delve into two additional strategies that can potentially lead to further distance savings in the generalized capacity-sharing vehicle routing problem with store transfers for PUP order delivery. Firstly, we explore the possibility of allowing partial demand fulfillment through transferring, where a portion of a store's PUP demand is transferred to the replenishment vehicle while the remaining demand is directly delivered. Secondly, we analyze the impact of permitting visits of multiple PUP vehicles to retail stores.

E.1 Effect of partial deliveries through transferring and through direct delivery on the distance savings

To illustrate the advantages of allowing partial deliveries through transferring and the remaining through direct visits to shared retail stores, we present an example scenario. Consider a case with 8 retail stores, out of which 6 are shared retail stores. Figure E.1 depicts the position of each retail stores. Each shared retail store has a PUP demand of 8, while the replenishment demand for each retail store is 10. The spare capacity of each replenishment vehicle for each replenishment route is 5. Table E.1 presents the distance matrix between the retail stores and warehouses.

c_{ij}	PW	RW	A	B	C	D	E	F	G	H
PW	-	28.08	17.26	43.57	57.08	45.01	49.40	56.08	63.03	61.85
RW	28.28	-	23.19	35.47	40.72	31.40	22.80	32.02	34.83	37.22

A	17.26	23.19	-	26.31	40.45	28.28	38.29	42.44	54.08	60.41
B	43.57	35.4	26.31	-	17.20	8.25	33.02	29.41	49.16	66.76
C	57.08	40.72	40.45	17.20	-	12.17	27.46	18.68	40.26	63.29
D	45.01	31.40	28.28	8.25	12.17	-	25.18	21.19	41.05	59.81
E	49.40	22.80	38.29	33.02	27.46	25.18	-	11.18	16.6	35.85
F	56.08	32.02	42.44	29.41	18.68	21.19	11.18	-	21.59	45.61
G	63.03	34.83	54.08	49.16	40.26	41.05	16.76	21.59	-	27.46
H	61.85	37.22	60.41	66.76	63.29	59.81	35.85	45.61	27.46	-

Table E.1: Distance matrix for example considering partial deliveries through transferring and direct delivery

E.1.1 Without partial transferring and partial direct delivery

Without allowing partial transferring and partial direct delivery to meet the PUP demand, the optimal routes of the PUP vehicles are $[PW \rightarrow A \rightarrow PW; PW \rightarrow B \rightarrow PW; PW \rightarrow C \rightarrow PW; PW \rightarrow D \rightarrow PW; PW \rightarrow G \rightarrow PW; PW \rightarrow H \rightarrow PW]$, and the total distance traveled is 575.59 units.

E.1.2 With partial transferring and partial direct delivery

Transfer store	A	B	C
Transferred store	C	C	D
Fraction of demand transferred	0.25	0.25	0.75
Demand transferred	2	2	6

Table E.2: Transfer set for example considering partial deliveries through transferring and direct delivery

By allowing partial transferring and partial direct delivery to meet the PUP demand, the optimal routes of the PUP vehicles are $[PW \rightarrow A \rightarrow PW; PW \rightarrow B \rightarrow PW; PW \rightarrow$

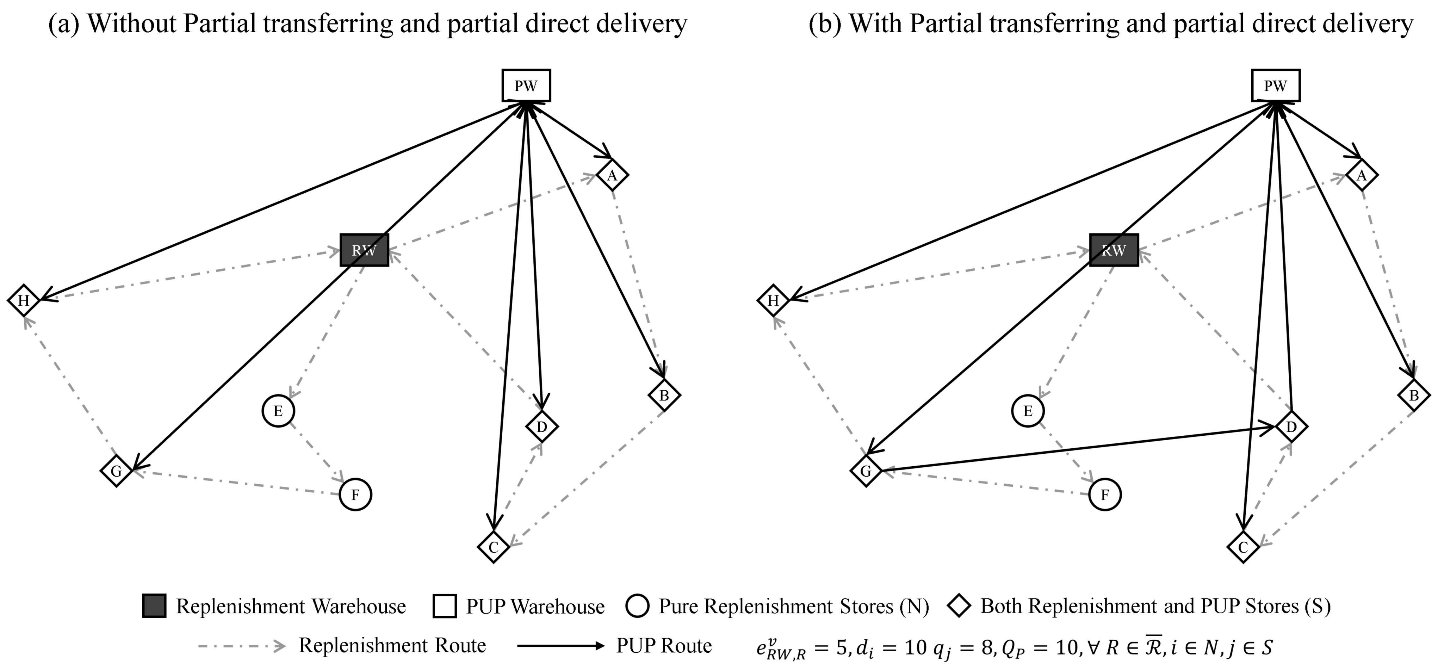

Figure E.1: Comparison of distance savings with and without partial deliveries through transferring and direct delivery: (a) without partial transferring and direct delivery, (b) with partial transferring and direct delivery

$D \rightarrow PW; PW \rightarrow G \rightarrow D \rightarrow PW; PW \rightarrow H \rightarrow PW]$, and the total distance traveled is 508.6 units. The transfer set is shown in table E.2.

In our MILP model, we have considered two options for fulfilling the PUP demand of shared retail stores: either fully transferring the demand at some previous retail stores or directly delivering the PUP demand to the shared retail stores. This is reflected in constraint (2.2) in section 2.3, i.e., $\sum_{\substack{i \in V \\ i \neq j}} x_{ij} + \sum_{\substack{i \in R \\ i < j}} z_{ij} = 1, \ \forall R \in \overline{\mathcal{R}}, j \in (S \cap R)$. Our model does not allow for partial transfer of the demand at previous stores and subsequent visit to fulfill the remaining demand at the shared retail stores.

However, in the specific example considered in this study, we observed that allowing partial demand fulfillment through transfer and the remaining demand fulfillment through direct visits resulted in more significant distance savings. The distance savings achieved were approximately 11.63%. This finding highlights the potential benefits of adopting a flexible approach that allows for partial transfers to optimize the routing and reduce overall delivery distances.

E.2 Effect of allowing multiple PUP vehicle visits to the retail stores on the distance savings

In this section, we investigate the benefits of allowing multiple visits of the PUP vehicle to the retail stores on the distance savings. To illustrate this, we consider an example where there are four retail stores, all of which also function as PUPs.

c_{ij}	PW	RW	A	B	C	D
PW	-	28.08	17.26	43.57	57.08	45.01
RW	28.28	-	23.19	35.47	40.72	31.40
A	17.26	23.19	-	26.31	40.45	28.28
B	43.57	35.4	26.31	-	17.20	8.25
C	57.08	40.72	40.45	17.20	-	12.17

D	45.01	31.40	28.28	8.25	12.17	-

Table E.3: Distance matrix for example considering multiple vehicle visits to the retail stores

The schematic representation of this example is depicted in figure E.2. Each retail store has a replenishment demand of 10 units and a PUP demand of 5 units. The replenishment vehicle at the replenishment warehouse has a spare capacity of 5 units. The distance matrix for this example can be found in table E.3. By analyzing this scenario, we aim to assess the impact of allowing multiple visits of the PUP vehicle on distance savings and overall routing efficiency.

E.2.1 Without multiple visits of PUP vehicles

Without allowing multiple visits of PUP vehicles to the retail stores to fulfill the PUP demand, the optimal routes of the PUP vehicles are $[PW \rightarrow A \rightarrow PW; PW \rightarrow B \rightarrow PW]$, resulting in a total distance traveled of 121.66 units. The transfer set for this scenario is presented in Table E.4.

Transfer store	A
Transferred store	D
Fraction of demand transferred	1
Demand transferred	5

Table E.4: Transfer set for example considering single visit of PUP vehicles to retail stores

E.2.2 With multiple visits of PUP vehicles

By allowing multiple visits of PUP vehicles to fulfill the PUP demand, the optimal routes of the PUP vehicles are $[PW \rightarrow A \rightarrow PW; PW \rightarrow A \rightarrow PW]$, resulting in a total distance traveled of 69.04 units. The transfer set for this scenario is provided in Table E.5. This represents a significant distance savings of approximately 43.25% compared to the previous case where multiple visits were not permitted.

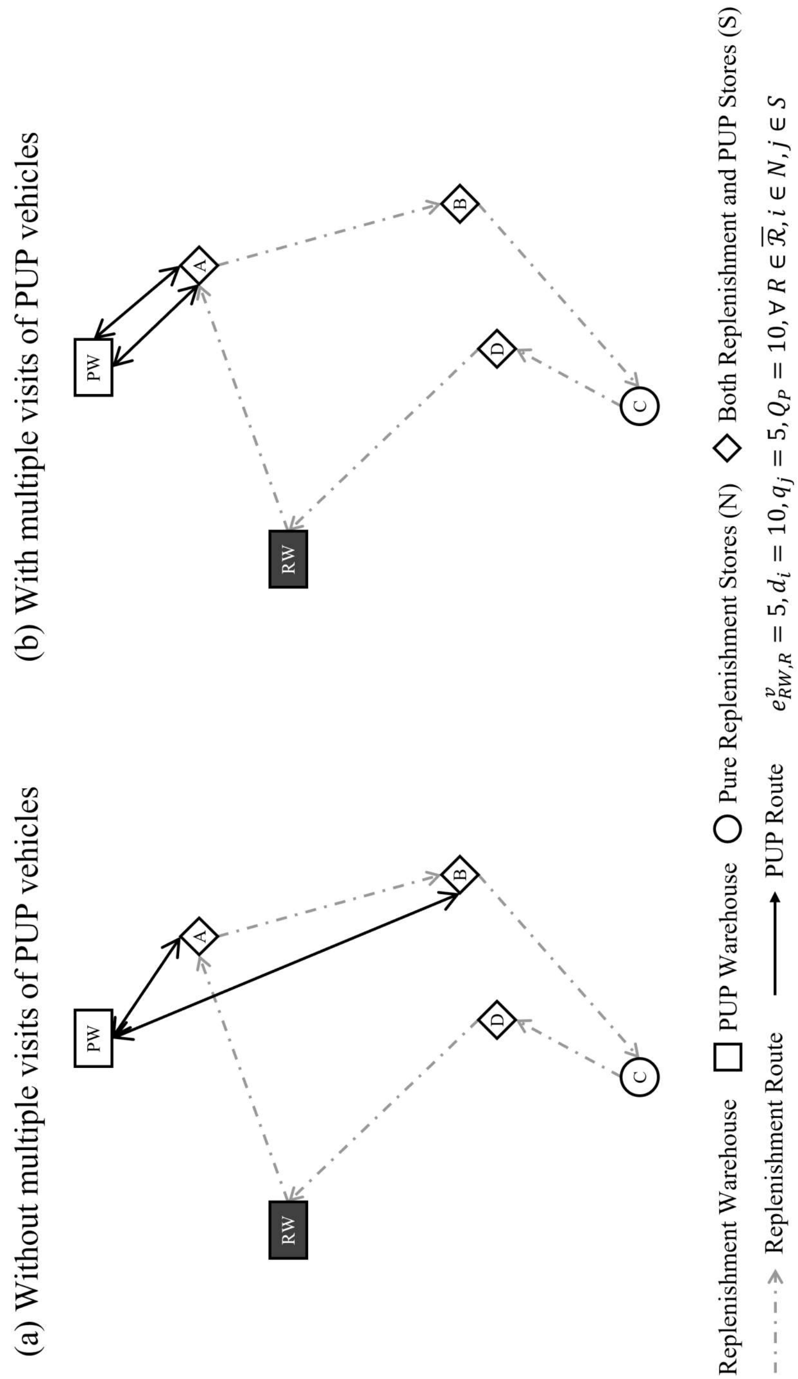

Figure E.2: Comparison of distance savings with and without multiple visits of PUP vehicles to the retail stores:(a) without multiple visits of PUP vehicles, (b) with multiple visits of PUP vehicles

Transfer store	A	A
Transferred store	D	B
Fraction of demand transferred	1	1
Demand transferred	5	5

Table E.5: Transfer set for example considering multiple visit of PUP vehicles to retail stores