

# Predictive Models for Time-to-Fail with Multiple Causes



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# Abstract

This thesis addresses the critical need for accurate failure time prediction for complex systems, with two real-life applications in industrial manufacturing and intensive care units (ICUs). In the first application, we developed a predictive model for analysing operational logs data from Injection Molding (IM) machine to model event durations and failure frequencies using exponential and truncated Poisson distributions respectively. Motivated by the need for accurate and interpretable diagnosis of IM machines used in plastic bottle manufacturing, a novel parametric framework for time-to-failure prediction based on operational log data. Machine operation is characterized by two latent states, ‘running without alert’ and ‘running with alert’, whose durations follow independent exponential distributions, while the number of alert events per operating epoch is modeled using a shifted Poisson distribution. The key contributions include the joint modeling of state-dependent failure dynamics and event frequency, with the proposed approach achieving performance comparable to the Cox proportional hazards model while preserving strong interpretability for predictive maintenance. The maximum likelihood estimation, with and without covariates, enables a predictive framework to optimize operational parameters and minimize the machine downtime.

In the second real application, we model ICU stay durations of several patients in Boston, MA, USA under a competing risks framework using the Medical Information Mart for Intensive Care (MIMIC-III) dataset. Each outcome, such as mortality or discharge, is treated as a latent failure time, with the observed duration determined by the earliest event. Parametric distributions including Weibull and Burr Type XII are evaluated for modeling these times. The inclusion of patient covariates (vitals, demographics, and history) allows for personalized risk estimation.

We propose a flexible piecewise linear polynomial for modeling the cumulative hazard with breakpoints estimated via a Non-Homogeneous Poisson Process. This model improves adaptability to irregular risk patterns. Rigorous validation using BIC for model selection, and MSE and Kaplan-Meier comparisons for assessing model fit, supports the appropriateness of the proposed models. Results offer actionable insights for both proac-

tive industrial maintenance and clinical decision-making, demonstrating the value of competing risks models in real-world applications.

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